Improving the Consensus Models For Group Decision-making Problems Based on Discrete Fuzzy Numbers

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***Abstract*—According to recent studies, discrete fuzzy numbers based on linguistic computing models gained great interest from researchers and practitioners due to their properties. However, the linguistic computing model's research regarding the group consensus is not enough and needs to be explored further. Besides, group consensus is substantial in making the optimized decision making. In this paper, we propose a novel consensus model based on discrete fuzzy numbers in a linguistic computing model that overcomes some of the main disadvantages of the previously proposed methods from the literature. In this work, we assimilated a new aggregation function and made a semi-automated algorithm that allows the experts to interact and modify their opinions during the simulation. This method achieves a greater rate of convergence and also a higher consensus degree in Group Decision-making problems.**

***Index Terms*—Consensus, Discrete Fuzzy Numbers, Aggregation Function, Linguistic Computing Models, Group Decision Making**

I. Introduction

The concept "consensus" is considered as a very essential term which means "settlement formed by the consent of several or all member of the group". The consensus covers fields such as judgment aggregation, social choice, and group decision making [1], [2]. Furthermore, in order to make an agreement, we used the method of consensus. This includes the advancement of the witnesses of members of the group to a "consensus" regarding their preliminary judgments. This process can be assisted by a distinct individual, who is identified as a moderator. Therefore, assumed the significance of gaining an acknowledged resolution by all individuals in the group, the consensus has achieved a prodigious consideration and it is fundamentally the main objective of group decision-making problems. Thus, there is a need for a whole consensus reaching process to achieve the final decision. For instance, the experts of a specific field try to reach a common solution or consensus, but each expert has his own preferences. The challenge here is, how they can reach a high level of agreement among them and take the final decision? The answer is: there is a need for a consensus reaching model that takes the preferences of the experts, treating them, measure and boost the consensus level between experts to achieve a satisfactory solution or consensus. Hence, many consensus models were proposed in the literature [3], [11], [12].

The most comprehensive methods among the best to achieve a consensus is centered on the expert's preferences, typically, over the supposed preference-relations. Owing to the problem's complexity, these preference-relations can be presented in different methods, numerous times inserted in a fuzzy environment [3]–[9].

Then, it was pertinent to inform that the discrete fuzzy numbers based model presents a few advantages as compared to other existing methods. Predominantly, in the discrete fuzzy numbers based linguistic computational modeling [16], the experts can assess the substitutions using distinct linguistic scales having better flexibility. In the linguistic models, the decision-makers should provide a single linguistic term from the set as an evaluation criterion. However, a single linguistic term is not expressed by the experts in most of the above studies. In order to address the complex problems, they used expressions such as "Good", "between fair" and "fair".

In order to develop the consensus ways for the group decision making (GDM) problems, the discrete fuzzy numbers based linguistic model was proposed in [10] and [18].

In [10] proposed a consensus model based on linguistic computing model which is able to attain consensus by maintaining the information and with no imposition of any drastic change. Along with, in [18], the authors propose a novel method to measure the consensus degree and an automatic algorithm to improve the consensus. The authors expose several advantages of their method with respect to the one proposed in [18] such as lower operating costs, better accuracy, better effectiveness, and rationality. In addition, among the novel features introduced in this paper, the use of weights on the initial preferences of the experts for aggregation purposes, the idea of discrete fuzzy number's uncertainty, and the automatic variations of the opinions to reach the consensus goal stand out.

Nevertheless, the Two methods that were presented in [10], [18], on some occasions, do not converge and therefore, a consensus is not reached. [18] has a major drawback with respect to the automation of the improvement of the consensus degree. Namely, the experts only take part at the beginning of the process, their opinions are changed without their approval and therefore, their individual final assessment may be different from their actual opinion. Experts must be allowed to take part during the whole process because, throughout the discussion, other experts can defend their positions and convince other experts to modify their opinions to other ones closer to those defended by themselves. Otherwise, it is not a true consensus process but an exploitation model to obtain an automatic final decision representative of the initial opinions. Finally, the method of improving group consensus in this paper do not work with all conditions within a decision making problems perhaps with specific conditions.

The main goals of this paper are to propose a novel consensus method based on discrete fuzzy numbers which combine the strong points of the two methods proposed in [10], [18] but also solves the main drawbacks such as the low rate of convergence, the high average number of required iterations and the fact that the experts do not play any role in the consensus process and their initial opinions are automatically changed without their validation.

The contributions of this work are as follow:

1) Propose an aggregation function on the set of discrete fuzzy numbers in line with [14], [15], [19]
2) Propose a new method to improve the consensus degree where it allows the experts to modify their opinions but that ensures the improvement of the consensus degree.
3) Constantly convergence and reach the highest consensus level

The rest of the paper is organized as follows, the second part devotes to the preliminaries, the third section introduces the proposed algorithm, Further an example will be presented in application. Then, discussion and finally, the conclusion.

II. Preliminaries

To understand better the paper, this section will be devoted to the main theories and definitions that will be used later.

By a fuzzy set of $\mathbb{R}$, we have a function A: $\mathbb{R} \rightarrow [0,1]$. For each fuzzy subset of A. let $A^\alpha = \{x \in \mathbb{R} : A(x) \geq \alpha\}$ for any $\alpha \in [0,1]$ be its α-level set (α-cuts). By Supp(A), we mean the support of A, i.e. the set of $\{x \in \mathbb{R} : A(x) > 0\}$. By A^0, we mean the closure of supp(A).

Definition 1 ([13]). *A fuzzy subset A of $\mathbb{R}$ with membership mapping $A : \mathbb{R} \rightarrow [0,1]$ is called discrete fuzzy numbers if its support is finite, i.e, there exist $x_1, x_2, \cdots, x_n \in \mathbb{R}$ with $x_1 < x_2 < \cdots < x_n$. Such that $Supp(A) = \{x_1, x_2, \cdots, x_n\}$, and there are natural numbers s, t with $1 \leqslant s \leqslant t \leqslant n$ such that:*

1. $A(x_i) = 1$ for any natural number i with $s \leqslant i \leqslant t$ (core)

2. $A(x_i) \leqslant A(x_j)$ for each natural number i,j with $1 \leqslant i \leqslant j \leqslant s$

3. $A(x_i) \geqslant A(x_j)$ for each natural number i,j with $t \leqslant i \leqslant j \leqslant n$

Henceforth, we will refer to the set of discrete fuzzy numbers as DFN and the discrete fuzzy number as dfn. Similarly, we will present by $A_1^{L_n}$ the set of discrete fuzzy numbers whose support is a sub-interval of the finite chain L_n.

$A, B \in A_1^{L_n}$ are two DFN. As the supports of A and B are sub-intervals of L_n thus are each one of its α-cuts. The α-level cuts for A and B are $A^\alpha = [x_1^\alpha, x_p^\alpha]$, $B^\alpha = [y_1^\alpha, y_p^\alpha]$.

Definition 2. *Let $A \in \mathcal{A}_1^{L_n}$ be a discrete fuzzy number. We will say that $\alpha \in (0,1]$ is a relevant α-level if there exists a $x \in Supp(A)$ such that $A(x) = \alpha$.*

Now let us introduce a method to generate aggregation functions on the set $\mathcal{A}_1^{L_n}$ by using discrete aggregation functions on L_n.

Theorem 3 ([14], [15]). *Consider a binary aggregation function F on the finite chain L_n. The binary operation on $A_1^{L_n}$ defined as follows*

$$F : A_1^{L_n} \times A_1^{L_n} \rightarrow A_1^{l_n}$$
$$(A, B) \rightarrow F(A, B)$$

being F(A,B) the dfn whose α-cuts are the sets:

$$\{z \in L_n | F(\min A^\alpha, \min B^\alpha) \leq z \leq F(\max A^\alpha, \max B^\alpha)\} \quad (1)$$

for each $\alpha \in [0,1]$ is an aggregation function on $A_1^{L_n}$.

Here, we remind the concept of linguistic model based on DFN whose support is an interval of the finite chain $L_n = \{0, 1, \cdots, n\}$.

Primary, we can think about a bijective mapping between the finite chain L_n and the ordinal scale $\sigma = \{s_0, \cdots, s_n\}$ by maintaining the original order. Secondly, each normal discrete convex fuzzy subset defined on the ordinal scale σ can be studied a a dfn in $A_1^{L_n}$. For example, let the linguistic hedge

$\sigma = \{EB, VB, B, MB, F, MG, G, VG, EG\}$

where the letters indicate the terms: Extremely Bad, Very Bad, Bad, More or less Bad, Fair, More or less Good, Good, Very Good and Extremely Good in the finite chain L_8 and they are mentioned in an increment order:

$EB < VB < B < MB < F < MG < G < VG < EG$

An an Example, the dfn

$A = \left\{\frac{0.8}{VB}, \frac{0.9}{B}, \frac{1}{F}, \frac{0.6}{G}\right\} \in A_1^{l_8}$ or

$A = \left\{\frac{0.8}{1}, \frac{0.9}{2}, \frac{1}{3}, \frac{0.6}{4}\right\} \in A_1^{l_8}$.

These fuzzy fuzzy subsets are a simple and flexible representations of the linguistic labels see Fig 1. Also, the values attached to the linguistic terms are named membership values.

Definition 4. *Let $L_n = \{0, \cdots, n\}$ be a finite chain. We recall a subjective evaluation to each dfn belonging to the partially ordered set $A_1^{L_n}$.*

As per recent observations, a subjective evaluation can be interpreted equivalently like a normal convex fuzzy subset defined on the original scale σ. A subjective evaluations with linguistic computational model was introduced in [16] with some interesting features [16], [17]. Hence, as a first introduced aspect : there is no need for any transformation or aggregation of information while the linguistic interpretation based on subjective evaluations. Each subjective evaluation defined by an expert is interpreted as a dfn of the set of $A_1 L_n$. Because of this, we can deal this information as per Theorem

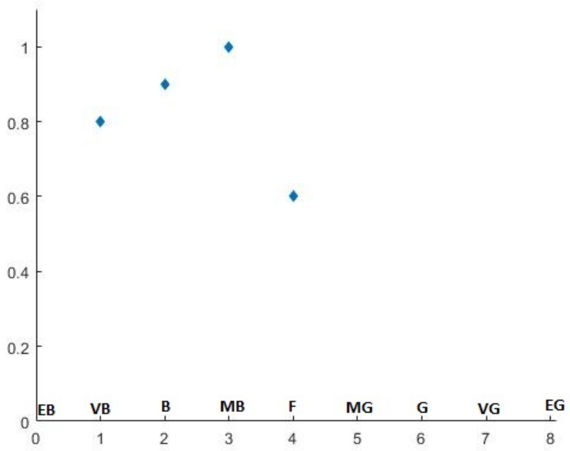

Fig. 1. Graphical Representation of subjective evaluation A

3. It is possible to define different flexibilizations of a linguistic expression through the following subjective evaluations [6] :

Between s_i and $s_j = \left\{A \in A_1^{Ln} | core\,(A) = [s_i, s_j]\right\}$

Worse than $s_i = \left\{A \in A_1^{Ln} | core\,(A) = [s_0, s_{i-1}]\right\}$

Better than $s_i = \left\{A \in A_1^{Ln} | core\,(A) = [s_{i+1}, s_n]\right\}$

for all $0 \leq i, j \leq n$. Hence, DFN $A \in A_1^{L_n}$ with $core\,(A) = [s_i, s_j]$, and with a different support, is considered as flexibilization of the subjective evaluation "between s_i and s_j, same with the other expressions. This Model allows to the experts to indicate their opinions in another format. In a manner, the experts are able now to employ linguistic scales with distinct granularity and to express a final decision that enclose all the evaluations in a different linguistic scales

III. Proposed Approach

The proposed approach is summarized in 5 main steps, see 2. These steps will be explained in the subsections below.

Initially, we presume that there is a group-evaluated problem. which has n number of experts. Where $A^1, A^2, \cdots, A^n$ are the evaluations in $\mathbb{R}$ and based on DFN. the subjective weights of experts are $w_s = \left\{w_s^1, w_s^2, \cdots, w_s^n\right\}$.

A. Computing the Combination Weights

The first step of the algorithm is denoted to the computational of the combination weights $w = \{w_1, w_2, \cdots, w_n\}$ in the same way as in [18].

Definition 5. *Let x_k represents the membership values of $A \in A_1^{L_n}$, m refers to the number of the linguistic terms and x is the membership values of A. The standard deviation of the membership values D(A) is defined by:*

$$D(A) = \frac{\sqrt{\sum_{k=1}^{m}(x_k - \sum_{k=1}^{m} \frac{x_k}{m})^2}}{m} \qquad (2)$$

Definition 6. *For dfn $A \in A_1^{L_n}$ where $L_n = \{0, 1, \cdots, n\}$, the uncertainty level is computed as follow:*

$$U(A) = \alpha\frac{a}{n} + (1 - \alpha)[1 - D(A)] \qquad (3)$$

Where a is the length of the dfn A interval, α is a value between 0 and 1 and D (A) is the standard deviation of the membership values defined in (2).

Definition 7. *The combination weights of w_s is computed as follow:*

$$w^p = \beta w_s^p + (1 - \beta)\frac{1 - U(A^p)}{\sum_{k=1}^{p}(1 - U(A^k))} \qquad (4)$$

where β is a value between 0 and 1 and U(A) is the uncertainty level defined in (3) .

Example: We have a groups of 4 experts where their evaluations:

$A^1 = \left\{\frac{1}{3}, \frac{1}{4}\right\}$

$A^2 = \left\{\frac{1}{0}, \frac{1}{1}, \frac{0.8}{2}\right\}$

$A^3 = \left\{\frac{0.1}{0}, \frac{0.4}{1}, \frac{1}{2}, \frac{0.4}{3}, \frac{0.3}{4}\right\}$

$A^4 = \left\{\frac{0.6}{1}, \frac{1}{2}, \frac{0.3}{3}\right\}$

A^1, A^2, A^3 and $A^4 \in A_1^{L_4}$ also, $\alpha = 0.5$, $\beta = 0.6$ and $w_s = \{0.3, 0.1, 0.5, 0.1\}$ We compute the Deviation by applying (2): $D(A^1) = \frac{\sqrt{\sum_{k=1}^{2}(x_k - \sum_{k=1}^{2} \frac{x_k}{2})^2}}{2} = \frac{\sqrt{\sum_{k=1}^{2}(x_k - 1)}}{2} = 0$, Then as well we obtained $D(A^2) = 0.41$, $D(A^3) = 0.81$, $D(A^4) = 0.37$

Later, we compute the Uncertainty by applying (3) :

$U(A^1) = \alpha\frac{a}{n} + (1 - \alpha)[1 - D(A^1)]$

$= 0.5\frac{1}{4} + (1 - 0.5)[1 - 0] = 0.625$. Similarly, $U(A^2) = 0.795$, $U(A^3) = 0.595$, $U(A^4) = 0.648$

Finally, we compute the combination weights using (4):

$w^1 = \beta w_s^1 + (1 - \beta)\frac{1 - U(A^1)}{\sum_{k=1}^{4}(1 - U(A^k))}$

$= 0.5 \times 0.3 + (1 - 0.5)\frac{1 - 0.625}{1.337} = 0.2922$

Similarly, $w^2 = 0.123, w^3 = 0.421$ and $w^4 = 0.165$

B. Computing the Aggregation Function

The second step of the algorithm is to aggregate the discrete fuzzy numbers with the combination weights.

Definition 8 ([19]). *Let $w = (w_1, w_2, \ldots, w_m)$ with $\sum_{i=1}^{i=m} w_i = 1$ a vector of combinational weights. The so-called* discrete weighted arithmetic mean *is given by*

$F : (L_n)^m \to L_n$
$F(x_1, \ldots, x_m) = \lceil W(x_1, \ldots, x_m) \rceil = \lceil \sum_{i=1}^{n} x_i w_i \rceil$.

Considering the discrete aggregation function introduced in Definition 8, by using Theorem 1, we can obtain an aggregation function on the set $\mathcal{A}_1^{L_n}$. We will denote by A^{ag} this aggregation function.

Example: We have two discrete fuzzy numbers $A^1 = \left\{\frac{1}{3}, \frac{1}{4}\right\}$ and $A^2 = \left\{\frac{1}{0}, \frac{1}{1}, \frac{0.8}{2}\right\}$. The combination weights are $w = \{0.5, 0.5\}$. The α-levels of A^1 and A^2 are $\{1, 0.8\}$.

A^1: 1-cuts = [3,4] and 0.8-cut=[3,4]

A^2: 1-cuts = [0,1] and 0.8-cut=[0,2]

We compute the α-levels of A_{ag} (agregation fuction) by applying the definition 6 and 7.

A^{ag}: 1-cuts $= \{z \in L_n / F(3, 0) \leq z \leq F(4, 1)\} = \{z \in L_n / \lceil 0.5 \times 3 + 0.5 \times 0 \rceil \leq z \leq \lceil 0.5 \times 4 + 0.5 \times 1 \rceil\} = [2, 3]$.

A^{ag}: 0.8-cuts $= \{z \in L_n / F(3,0) \leq z \leq F(4,2)\} = \{z \in L_n / \lceil 0.5 \times 3 + 0.5 \times 0 \rceil \leq z \leq \lceil 0.5 \times 4 + 0.5 \times 2 \rceil\} = [2,3]$.

$\Rightarrow A_{ag} = \left\{\frac{1}{2}, \frac{1}{3}\right\}$

C. Computing Group Consensus Index

To compute the index (score) of the Consensus in a group we need to apply the definition 9 proposed in [18].

Definition 9. *Let $A^1, A^2, \ldots, A^m \in \mathcal{A}_1^{L_n}$ be the opinions of m experts and $w = \{w^1, w^2, \cdots, w^t\}$ with $\sum_{i=1}^{i=m} w_i = 1$ be a vector of combinational weights. Let A^{ag} the result of the aggregation of $A^1, A^2, \ldots, A^m$. The group consensus index is computed by:*

$$GC(A_1, \ldots, A_m) = \sum_{l=1}^{m} w_l (1 - D_{l,ag}) \tag{5}$$

where $D_{l,ag}$ is the deviation between the opinions given by

$$\begin{aligned} D_{l,ag} &= \lambda \left[\frac{1}{n(r+1)} \sum_{i=0}^{r} |p_{l,i} - p_{ag,i}| \right] \\ &+ (1-\lambda) \left[\frac{1}{n(r+1)} \sum_{i=0}^{r} |n_{l,i} - n_{ag,i}| \right] \end{aligned} \tag{6}$$

where $\lambda \in [0,1]$, $n_{l,i}$ and $n_{ag,i}$ are the lower and $p_{l,i}$ and $p_{ag,i}$ are the upper limits of the α-cuts of A_l and A^{ag}, respectively. To compute this, we consider the union of the relevant α-levels of $A^1, A^2, \ldots, A^m$.

D. Computing the Distance

To compute the distances between the DFN $A^1, A^2, \ldots, A^m$ and the aggregation functionA_{ag}, we apply the equation in definition 10 [6].

Definition 10. *Given any two discrete fuzzy numbers A and B $\in A_1 l_n$, the distance between A and B is computed by applying the equation below:*

$$d(A,B) = \frac{1}{2k(n+1)} \sum_{i=1}^{k} \alpha_i (|a_1^{\alpha_i} - b_1^{\alpha_i}| + |a_2^{\alpha_i} - b_2^{\alpha_i}|) \tag{7}$$

Where $[a_1^{\alpha_i}, a_2^{\alpha_i}]$ and $[b_1^{\alpha_i}, b_2^{\alpha_i}]$ are the α-cuts of A and B (intervals) $0 \leq alpha_i \leq 1$ with $1 \leq i \leq$k (k: different levels)

E. Improving Group Consensus

In order to improve the Group Consensus, we proposed an algorithm which the experts can intervene to modify their preferences. The steps of algorithm are summarized as follow:

- Select the highest computed distance that corresponds to one expert preference.

- Random Process: Summarized as follow:

1. Generate N random DFN where for each one random DFN, we will update our DFN input values from the GDM problem based on the previous condition

2. Repeat the process from step 3 till reaching the consensus

3. Store all the converged random DFN

- Propose to the selected expert S number of preferences (from the stored one) and ask the expert to choose between which one that correspond near to his own opinion.

- Repeat the previous step till the expert find the preferences near to his own opinions

F. Application

In the application, we present the example as of the Xiao paper [18].

We take group of 4 experts as input of an input which are given below:

Example: We have a groups of 4 experts where their evaluations:

$A^1 = \left\{\frac{0.6}{2}, \frac{1}{3}, \frac{0.7}{4}\right\}$
$A^2 = \left\{\frac{0.6}{3}, \frac{1}{4}, \frac{0.8}{5}, \frac{0.7}{6}\right\}$
$A^3 = \left\{\frac{0.4}{5}, \frac{1}{6}, \frac{0.9}{7}\right\}$
$A^4 = \left\{\frac{0.6}{5}, \frac{1}{6}, \frac{0.8}{7}, \frac{0.7}{8}\right\}$

Furthermore, the other necessary inputs are as follow:

$\alpha = 0.5$ $\beta = 0.6$ $w_s = \{0.3, 0.1, 0.5, 0.1\}$

$r = 10; \lambda = 0.5; n = 8; \theta = 0.9$

where, w_s are the subjective weights.

Now, initial inputs are setup. We shall present each step implemented with the results.

Step 1 Combination Weights

According to the definition 3 and Equ. (2), we calculated the standard deviation for all the experts $A_1 ... A_4$. The results are as follow.

D (A_1) = 0.17, D (A_2) = 0.15, D (A_3) = 0.26, D (A_4) = 0.15

Then, According to the definition 4 and Equ. (3), we calculated the uncertainty for all the experts $A_1 ... A_4$. The results are as follow.

U (A_1) = 0.54, U (A_2) = 0.61, U (A_3) = 0.49, U (A_4) = 0.61

In order to compute the combination weights, we used the Equ. (4) as per definition 5. The combination weights results are: W_1 = 0.286, W_2 = 0.149, W_3 = 0.416, W_4 = 0.149

Step 2 Aggregation Results

In this step, first we computed the $\alpha - levels$ as per definition 6.

$\alpha - levels$ = $1, 0.9, 0.8, 0.7, 0.6, 0.4$ Then, we calculated the $\alpha - cuts$ according to definition 6. The results are presented below.

A^1: 1-cuts = [3,3], 0.9-cut=[3,3], 0.8-cuts = [3,3], 0.7-cuts = [3,4], 0.6-cut=[2,4] and 0.4-cut=[2,4].

A^2: 1-cuts = [4,4], 0.9-cut=[4,4], 0.8-cuts = [4,5], 0.7-cuts = [4,6], 0.6-cut=[3,6] and 0.4-cut=[3,6].

A^3: 1-cuts = [6,6], 0.9-cut=[6,7], 0.8-cuts = [6,7], 0.7-cuts = [6,7], 0.6-cut=[6,7] and 0.4-cut=[5,7].

A^4: 1-cuts = [6,6], 0.9-cut=[6,6], 0.8-cuts = [6,7], 0.7-cuts = [6,8], 0.6-cut=[5,8] and 0.4-cut=[5,8].

Finally, after calculating the $\alpha - level$ and $\alpha - cut$. We applied the weights and aggregated results are as follow:

$\Rightarrow A_{ag} = \left\{\frac{0.6}{4}, \frac{1}{5}, \frac{0.9}{6}, \frac{0.7}{7}\right\}$

Step 3 : Group Consensus Index

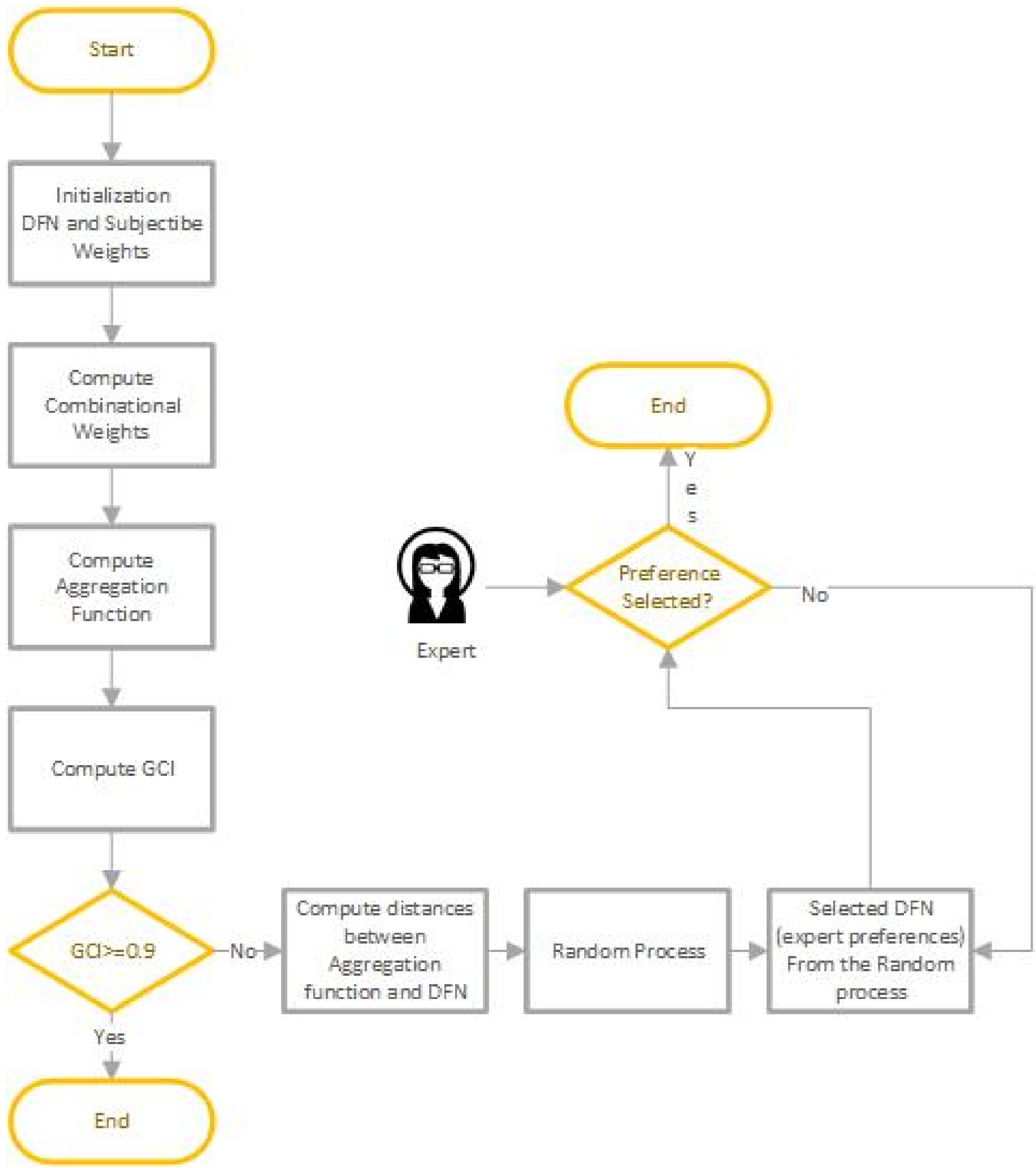

Fig. 2. Proposed Approach

In order to calculate the group consensus index, we have to calculate the deviation among each fuzzy discrete number $(A_1, ..., A_4)$ and aggregated result (A_{ag}) according to Equ. (6) in definition 8. The results of the deviation as follow:

$D_{1,ag}$ = 0.31, $D_{2,ag}$ = 0.13, $D_{3,ag}$ = 0.09, $D_{4,ag}$ = 0.12

Then, we used the Equ. (5) to calculate the group consensus index and result of is given below.

GCI = 0.8372

where, GCI is the group consensus index.

since, the GCI is less than $\theta = 0.9$. which means it is not converged. Therefore, we need to compute the distance. If the GCI is converged, then, we do not need to apply further.

Step 4 : Computing Distance

According to the Equ. (7) and definition 10. We calculated the distance among each fuzzy discrete number $(A_1, ..., A_4)$ and aggregated result (A_{ag}) d(A_{ag}, A_1)= 0.47, d(A_{ag}, A_2)= 0.22, d(A_{ag}, A_3)= 0.15, d(A_{ag}, A_4)= 0.18

Step 5 : Improving Group Consensus Index

Now, we have to select the expert with the highest distance which is d(A_{ag}, A_1)= 0.47. Then, as per our algorithm, experts is invoked to alter preferences. In order to facilitate the expert, our proposed algorithm generated 50 random experts and kept replacing the highest distance expert with one of the fifty random experts while holding the rest 3 experts the same. Every time, combination weights, aggregation results, and group consensus are calculated. Our proposed algorithm presented random experts who are greater or equal to the previous consensus result. Then, the highest distance expert got offered to exchange the preferences. Once the offer is selected by expert.

We found the updated experts, which is given below:
$A^1 = \left\{ \frac{0.1}{2}, \frac{1}{3}, \frac{0.6}{4}, \frac{0.2}{5}, \frac{0.3}{6} \right\}$
$A^2 = \left\{ \frac{0.6}{3}, \frac{1}{4}, \frac{0.8}{5}, \frac{0.7}{6} \right\}$
$A^3 = \left\{ \frac{0.4}{5}, \frac{1}{6}, \frac{0.9}{7} \right\}$
$A^4 = \left\{ \frac{0.6}{5}, \frac{1}{6}, \frac{0.8}{7}, \frac{0.7}{8} \right\}$
We observed that the expert A^1 is replaced. Furthermore, we went to step 3. In addition, we found that the updated group consensus index $GCI = 0.8464$ in first iteration. However, as it is not converged, then the algorithm remains processing till it converged.

In our case, the group consensus was converged in 2 iterations. The group consensus index in the 2 iterations with the initial group consensus are as follow:

Initial GCI = 0.8372, First Iteration GCI = 0.8464

Second Iteration GCI = 0.916

Discussion

In this paper, we have developed an algorithm which has a strong ability to develop the group consensus. Our proposed algorithm can converge efficiently as compared to the existing literature such as [18]. In order to validate our proposed algorithm, we applied the same example which is being studied in the [18]. Our algorithm performed efficiently and converged the group consensus index quickly with only two iterations. However, If we observe the results in [18], the group consensus index is converged at tenth iterations. This shows the effectiveness of our proposed algorithm.

In order to further validate our algorithm. We generated the 1000 groups of 4 discrete random numbers with $n = 4$, $n = 6$ and $n = 8$. We applied our proposed algorithm and algorithm in [18] on these 1000 groups with $n = 4$, $n = 6$ and $n = 8$ to compare the convergence rate and average iteration. Table I presents the simulation results which shows that our proposed algorithm outperforms the algorithm presented in the [18]. With $n = 4$, it is observed that Xiao algorithm [18] has 35.4% convergence rate whereas our proposed algorithm is 98.7% efficient. Moreover, we noticed that our proposed algorithm and algorithm in [18] have more or less similar computation time. Furthermore, we can see that our algorithm is presenting outstanding results even with the $n = 6$ and $n = 8$. The proposed algorithm has the capacity to deal with conflicts and develop resolutions among the group members in order to do effective group decisions.

Conclusion

As we know that group consensus plays an essential role in making effective decision making. we explore the linguistic computing model in order to study the group consensus application. In this paper, we proposed the innovative consensus model which is based on the discrete fuzzy numbers in a linguistic computing model. In addition, we developed a new aggregation function to aggregate the discrete fuzzy experts. Moreover, we also presented the semi-automated algorithm which is proved interactive for the experts and give the ability to the expert to modify opinion on the run time. According to the results, our proposed algorithm outperformed all the existing methods in the literature with the group consensus convergence rate 98.7% at $n = 4$, 99.9% at $n = 6$, and 100% at $n = 8$. This proposed algorithm is useful for solving the decision making problems in every walk of life.

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	Xiao Algorithm [18]		Our Proposed Algorithm	
	Convergence(%)	Iterations (Average)	Convergence(%)	Iterations (Average)
n=4	35.4	3	98.7	5
n=6	23.5	3	99.9	4
n=8	15.7	3	100	3

TABLE I

COMPARISON OF OUR METHOD WITH OTHER STUDIES.

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