An Area Similarity Algorithm Based CWW Model and Its Application to Service Quality Evaluation

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***Abstract*—Service quality evaluation is the beginning of making continuous improvements in providing service to consumers. However, the existing evaluation methods mostly model evaluation information by crisp number or Type-1 fuzzy set (T1 FSs), which cannot effectively reflect the uncertainty of users' perception. In this paper, a Computing With Words (CWW) model based on an area similarity algorithm of interval type-2 fuzzy sets (IT2 FSs) is constructed and applied to service quality evaluation. First, an area similarity measure algorithm is proposed to calculate the similarity between trapezoidal IT2 FSs. With the area similarity measure, a CWW model is developed using the similarity measure as CWW engine. The CWW model is then applied to a public transport service evaluation problem to sort each evaluation dimension to a class. The comparative analysis shows that our method can give more separated classifying results, which means a larger amount of information is provided to decision-makers.**

***Keywords*—Computing with words (CWW), area similarity measure, service evaluation, IT2 FS, TOPSIS**

I. INTRODUCTION

With the fast pace of globalization, competition between firms that offer substitute products has become more and more fierce. In order to gain as much profit in this competitive environment, it is not enough through only price strategy [1]. For service industry whose products are service rather than tangible goods, companies must offer satisfying service so as to attract and retain a large number of customers.

Service quality evaluation is the beginning of improving service quality. It aims to measure the degree of customer satisfaction [2], and then promotes continuous improvement, especially focusing on the weak aspects. Unlike commercial goods, service is intangible, cannot be stored and disappears as it is consumed [3]. Considering these unique characteristics, service evaluation heavily depends on the perception of users. The existing service evaluation methods can be classified into three categories: case study [4, 5], statistical analysis [6, 7] and multi-attribute decision analysis (MADA).

Some researchers investigate service evaluation problem in the framework of MADA [8-12]. Awasthi et al. [13] designed a questionnaire based on SERVQUAL to collect linguistic assessments and then transformed the assessments to triangular T1 FSs that are fused through TOPSIS. Chou et al. [14] established a fuzzy weighted SERVQUAL model to evaluate the airline service quality, where T1 FSs are used to model linguistic terms. Perçin [15] designed an integrated approach based on DEMATEL, ANP and VIKOR for airline service evaluation under T1 FS environment. However, these methods fail to reflect and handle the subjectivity and ambiguity of the evaluation provided by consumers. They transformed consumers' evaluation (words) to crisp numbers or T1 FSs, which leads to a large amount of useful information being lost. Because words mean different things to different people, a crisp number or a T1 FS with crisp membership grade cannot well represents a word. Compared to the above two forms, IT2 FS can reserve most subjectivity and ambiguity with its interval membership grade. And the information will be reserved and propagated to final evaluation results. Therefore, in this paper linguistic ratings are converted to IT2 FSs processed in the computing with words (CWW) model.

CWW [16,17,18] is a methodology which can directly process words and propositions extracted from natural language. The advantage of CWW compared to traditional operation on crisp number is that it is able to reflect and reserve the uncertainty hidden in the minds of human. The way of processing words is firstly transforming words into fuzzy sets and then performing operation on fuzzy sets. Due to the capacity of interval type-2 fuzzy sets (IT2 FSs) on modeling both inter-personal and intra-personal uncertainty [18], IT2 FSs are chosen as the fuzzy set operated in the CWW. CWW is able to overcome the drawbacks of the existing service evaluation methods, so we developed a service evaluation model under the framework of CWW. There are two types of CWW engine, i.e. linguistic weighted average (LWA) [19] and perceptual reasoning (PR) [20, 21]. Notwithstanding the two existing CWW engines, sometimes they cannot meet the requirement of CWW problems. When there are several alternatives to be chosen or evaluated, one way is to compare each alternative to the ideal best solution and choose the one that is closest to the ideal solution [22].

Inspired by that, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is integrated in the CWW model acting as CWW engine. The core of TOPSIS is the distance measure between each solution and the ideal solutions. In some existing works, Euclidean distance of triangular T1 FSs was used to calculate the distance between each alternative and two ideal solutions [11,23]. Arslan [24] modified standard Euclidean distance through introducing Weber-Fechner psycho-physical law. Celik et al. [10] used grey relational coefficient to measure the closeness of each alternative to ideal solutions. However, when IT2 FS is used to represent evaluation, there is no universal distance measure for it. Ali et al. [25] chose to defuzzify IT2 FS to crisp number and then employed the standard Euclidean distance. However, the defuzzification brings about information loss and hence the transformation from words to IT2 FSs to reserve more uncertainty becomes meaningless. In this paper, we substitute "distance" with "similarity" and slightly modify standard TOPSIS to accommodate the problem. Inspired by the Jaccard similarity measure, we proposed an area similarity measure for IT2 FSs and derive the detailed algorithm aimed for trapezoidal IT2 FSs.

In summary, our CWW model for service evaluation has following three major contributions. (1) To our knowledge, this is the first try to employ CWW paradigm to solve service evaluation problem. CWW's ability of processing words well accommodates service evaluation problem's heavy dependence on consumers' judgement. (2) Linguistic ratings collected from consumers are transformed to IT2 FSs processed in the CWW model. IT2 FS can reserve the uncertainty in evaluation information to a larger extent compared with crisp value and T1 FS. (3) We propose an area similarity measure for IT2 FSs, which performs CWW engine in the CWW model. The area-based similarity measure quantifies the similarity between two IT2 FSs from geometrical perspective, which can save most effective information.

The remainder of this paper is organized as follows. In section 2, some preliminary knowledge about CWW, trapezoidal IT2 FS and Jaccard similarity measure are introduced. In section 3, the area similarity measure for trapezoidal IT2 FSs is proposed and tested on a 32-IT2 FS database. In section 4, a CWW evaluation model is built, which is based on the area similarity measure proposed in section 3. In section 5, the CWW model is applied to a public transport service evaluation problem to demonstrate its practicability. Finally, Section 6 draw some conclusions.

II. PRELIMINARIES

In this section, we present some background knowledge about CWW and Per-C, trapezoidal IT2 FS and Jaccard similarity measure to help readers better understand the subsequent method section.

A. CWW and Perceptual Computer (Per-C)

Per-C [17] is a specific architecture of CWW. It is composed of three parts: encoder, CWW engine and decoder, as shown in Fig.1. Encoder is designed to convert words to fuzzy sets and its output is a codebook where each word is associated with a fuzzy set. This process can be achieved through Interval Approach (IA) [26] and Person FOU [27], etc. The fuzzy sets produced by encoder then activate the CWW engine and are processed to other fuzzy sets, which are then delivered to decoder. Decoder decodes the fuzzy sets to something that can be directly understood by human.

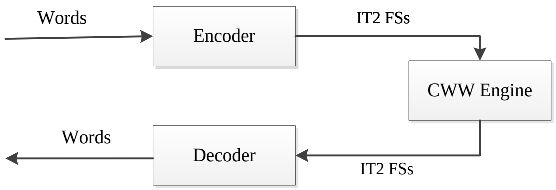

Fig. 1. The Structure of Perceptual Computer (Per-C)

B. Trapezoidal IT2 FSs

Suppose there are two trapezoidal IT2 FSs $\tilde{A}$ and $\tilde{B}$, as shown in Fig.2. $\tilde{A}$ is denoted as $\tilde{A}=\left[\tilde{A}^U,\tilde{A}^L\right]=[(a_1^U,a_2^U,a_3^U,a_4^U),(a_1^L,a_2^L,a_3^L,a_4^L,h_A)]$, where $\tilde{A}^U=(a_1^U,a_2^U,a_3^U,a_4^U)$ denotes the upper trapezoidal T1 FS, while $\tilde{A}^L=(a_1^L,a_2^L,a_3^L,a_4^L,h_A)$ denotes the lower trapezoidal T1 FS. The height of $\tilde{A}^U$ is 1, and the height of $\tilde{A}^L$ is h_A. Similarly, $\tilde{B}$ is denoted as $\tilde{B}=\left[\tilde{B}^U,\tilde{B}^L\right]=[(b_1^U,b_2^U,b_3^U,b_4^U),(b_1^L,b_2^L,b_3^L,b_4^L,h_B)]$. The height of $\tilde{B}^U$ is 1, and the height of $\tilde{B}^L$ is h_B. The upper membership function(UMF) and lower membership function(LMF) of $\tilde{A}$ and $\tilde{B}$ are as follows. $\overline{\mu}_{\tilde{A}}(x)$ and $\underline{\mu}_{\tilde{A}}(x)$ denote the UMF and LMF of $\tilde{A}$, respectively. $\overline{\mu}_{\tilde{B}}(x)$ and $\underline{\mu}_{\tilde{B}}(x)$ denote the UMF and LMF of $\tilde{B}$, respectively.

$$\overline{\mu}_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1^U}{a_2^U-a_1^U} & a_1^U<x\leqslant a_2^U\\ 1 & a_2^U<x<a_3^U\\ \dfrac{a_4^U-x}{a_4^U-a_3^U} & a_3^U\leqslant x<a_4^U\\ 0 & otherwise\end{cases} \tag{1}$$

$$\underline{\mu}_{\tilde{A}}(x)=\begin{cases}\dfrac{h_A(x-a_1^L)}{a_2^L-a_1^L} & a_1^L<x\leqslant a_2^L\\ h_A & a_2^L<x<a_3^L\\ \dfrac{h_A(a_4^L-x)}{a_4^L-a_3^L} & a_3^L\leqslant x<a_4^L\\ 0 & otherwise\end{cases} \tag{2}$$

$$\overline{\mu}_{\tilde{B}}(x)=\begin{cases}\dfrac{x-b_1^U}{b_2^U-b_1^U} & b_1^U<x\leqslant b_2^U\\ 1 & b_2^U<x<b_3^U\\ \dfrac{b_4^U-x}{b_4^U-b_3^U} & b_3^U\leqslant x<b_4^U\\ 0 & otherwise\end{cases} \tag{3}$$

$$\underline{\mu}_{\tilde{B}}(x)=\begin{cases}\dfrac{h_B(x-b_1^L)}{b_2^L-b_1^L} & b_1^L<x\leqslant b_2^L\\ h_B & b_2^L<x<b_3^L\\ \dfrac{h_B(b_4^L-x)}{b_4^L-b_3^L} & b_3^L\leqslant x<b_4^L\\ 0 & otherwise\end{cases} \tag{4}$$

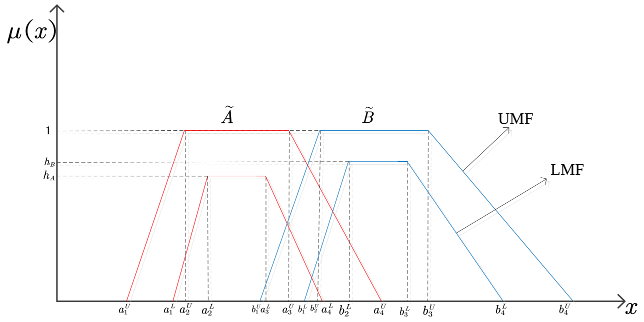

Fig. 2. Two trapezoidal IT2 FSs $\tilde{A}$ and $\tilde{B}$.

C. Jaccard similarity measure

Jaccard similarity for T1 FS defines the similarity as the ratio of cardinality of intersection on union of two T1 FSs. Wu [28] defined the average cardinality of an IT2 FS and then extended the Jaccard similarity measure from T1 FSs to IT2 FSs. It is calculated as (5).

$$sm_J\left(\tilde{A},\tilde{B}\right)=\frac{AC\left(\tilde{A}\bigcap\tilde{B}\right)}{AC\left(\tilde{A}\bigcup\tilde{B}\right)}$$
$$=\frac{\int_X \min\left(\bar{\mu}_{\tilde{A}}(x),\bar{\mu}_{\tilde{B}}(x)\right)dx+\int_X \min\left(\underline{\mu}_{\tilde{A}}(x),\underline{\mu}_{\tilde{B}}(x)\right)dx}{\int_X \max\left(\bar{\mu}_{\tilde{A}}(x),\bar{\mu}_{\tilde{B}}(x)\right)dx+\int_X \max\left(\underline{\mu}_{\tilde{A}}(x),\underline{\mu}_{\tilde{B}}(x)\right)dx} \quad (5)$$

$sm_J\left(\tilde{A},\tilde{B}\right)$ denotes the similarity between two IT2 FS $\tilde{A}$ and $\tilde{B}$ (Fig.2), $AC(\bullet)$ denotes the cardinality of an IT2 FS. $\tilde{A}\bigcap\tilde{B}$ and $\tilde{A}\bigcup\tilde{B}$ are the intersection and union of $\tilde{A}$ and $\tilde{B}$, respectively. $\bar{\mu}_{\tilde{A}}(x)$ and $\underline{\mu}_{\tilde{A}}(x)$ are the upper and lower membership function of $\tilde{A}$, respectively. Similarly, $\bar{\mu}_{\tilde{B}}(x)$ and $\underline{\mu}_{\tilde{B}}(x)$ are the upper and lower membership function of $\tilde{B}$, respectively.

III. An Area-Based Similarity Measure for Trapezoidal IT2 FSs

Inspired by the Jaccard similarity measure for trapezoidal IT2 FSs, we proposed an area-based similarity measure. First, we give the equation and algorithm to compute it. Then the proposed similarity measure is applied to a 32-IT2FSs database to verify its validity.

A. The proposed area-based similarity for trapezoidal IT2 FS

The main idea of our area similarity measure is the similarity of two IT2 FSs equals to ratio of the area of intersection part to the union part. The equation for it is as follows.

$$sm_J\left(\tilde{A},\tilde{B}\right)=\frac{S_{\bar{A}\wedge\bar{B}}+S_{\underline{A}\wedge\underline{B}}}{S_{\bar{A}\vee\bar{B}}+S_{\underline{A}\vee\underline{B}}} \quad (6)$$

where $S_{\bar{A}\wedge\bar{B}}$ is the area of intersection part of two UMFs, and $S_{\underline{A}\wedge\underline{B}}$ is the area of intersection part of two LMFs. $S_{\bar{A}\vee\bar{B}}$ is the area of union of two UMFs, and $S_{\underline{A}\vee\underline{B}}$ is the area of union of two LMFs.

Eq. (6) can be derived to Eq. (7).

$$sm_J\left(\tilde{A},\tilde{B}\right)=\frac{S_{\bar{A}\wedge\bar{B}}+S_{\underline{A}\wedge\underline{B}}}{S_{\bar{A}}+S_{\bar{B}}-S_{\bar{A}\wedge\bar{B}}+S_{\underline{A}}+S_{\underline{B}}-S_{\underline{A}\wedge\underline{B}}} \quad (7)$$

$S_{\bar{A}}$ and $S_{\underline{A}}$ denote the area closed by UMF and LMF of $\tilde{A}$, respectively. $S_{\bar{B}}$ and $S_{\underline{B}}$ denote the area closed by UMF and LMF of $\tilde{B}$, respectively. Next, we present the algorithm for the calculation of the six individual parts $S_{\bar{A}}$, $S_{\underline{A}}$, $S_{\bar{B}}$, $S_{\underline{B}}$, $S_{\bar{A}\wedge\bar{B}}$ and $S_{\bar{A}\vee\bar{B}}$.

B. The calculation algorithm for the area similarity measure

1) Calculation of $S_{\bar{A}}$, $S_{\underline{A}}$, $S_{\bar{B}}$ and $S_{\underline{B}}$

$S_{\bar{A}}$, $S_{\underline{A}}$, $S_{\bar{B}}$ and $S_{\underline{B}}$ are easily calculated because they denote the area closed by the four trapezoidal membership functions of $\tilde{A}$ and $\tilde{B}$, respectively. They can be calculated as long as we know the parameters of $\tilde{A}$ and $\tilde{B}$. The results are as follows.

$$S_{\bar{A}}=\int_x \bar{\mu}_{\tilde{A}}(x)dx=\frac{1}{2}\left[\left(a_3^U-a_2^U\right)+\left(a_4^U-a_1^U\right)\right] \quad (8)$$

$$S_{\underline{A}}=\int_x \underline{\mu}_{\tilde{A}}(x)dx=\frac{h_A}{2}\left[\left(a_3^L-a_2^L\right)+\left(a_4^L-a_1^L\right)\right] \quad (9)$$

$$S_{\bar{B}}=\int_x \bar{\mu}_{\tilde{B}}(x)dx=\frac{1}{2}\left[\left(b_3^U-b_2^U\right)+\left(b_4^U-b_1^U\right)\right] \quad (10)$$

$$S_{\underline{B}}=\int_x \underline{\mu}_{\tilde{B}}(x)dx=\frac{h_B}{2}\left[\left(b_3^L-b_2^L\right)+\left(b_4^L-b_1^L\right)\right] \quad (11)$$

2) Calculation algorithm for $S_{\bar{A}\wedge\bar{B}}$ and $S_{\underline{A}\wedge\underline{B}}$

$S_{\bar{A}\wedge\bar{B}}$ denotes the area of intersection of two UMFs, while $S_{\underline{A}\wedge\underline{B}}$ denotes that of two LMFs. The difference is that two UMFs have equal height, both equal to 1, while the heights of two LMFs are not equal, denoted as h_A and h_B, respectively.

Babak[29] provided a method to calculate the area of intersection of two normal trapezoidal T1 FSs whose heights are equal to 1. We adopt this idea and extend it to trapezoidal IT2 FSs. The main procedure of our algorithm is as follows.

a) List all the possible points that comprise the intersection polygon. For two UMFs, there are ten possible intersection points, denoted as $\bar{p}_1,\bar{p}_2,\ldots,\bar{p}_{10}$, which are the vertices of the intersection polygon. For two LMFs, there are also ten possible points, denoted as $\underline{p}_1,\underline{p}_2,\ldots,\underline{p}_{10}$, respectively. How to obtain the twenty possible intersection points will be introduced afterwards.

b) Check if the ten points are real intersection points. If one point $p_0=(x_0,y_0)$ is an intersection point, it should satisfy $0\leqslant y_0\leqslant 1$ and $\mu_{\tilde{A}}(x_0)=\mu_{\tilde{B}}(x_0)$.

c) Calculate area of the intersection polygon. Suppose there remain n valid points passed check in procedure *b)*

$(0 \leqslant n \leqslant 6)$. Order them in counter-clock fashion and re-denote as $p'_1, p'_2, \ldots, p'_n$. These n points compose the intersection polygon. Then calculate the area of the polygon closed by these n points according to Eq. (12).

$$Area = \frac{1}{2}\left[y_1(x_n - x_2) + y_2(x_1 - x_3) + y_3(x_2 - x_4) + \ldots + y_n(x_{n-1} - x_1)\right] \tag{12}$$

Now we analyze how to determine the ten possible points comprising the intersection polygon of two UMFs and LMFs mentioned in step *a)*, respectively.

3) Ten possible points comprising the intersection polygon of two UMFs

The ten possible intersection points (Fig.3) can be divided into three groups. Two upper horizontal sides of two trapezoidal UMFs have four possible intersection points, denoted as $\overline{p}_1$ to $\overline{p}_4$. Two points are located in x-axis, denoted as $\overline{p}_5$ and $\overline{p}_6$. Though these two points are not intersection points, they are inevitable points composing the intersection polygon and the validity of these two points do not need to check. The left and right shoulder of two UMFs have four possible intersection points, denoted as $\overline{p}_7$ to $\overline{p}_{10}$. $\overline{p}_7$ denotes the intersection points of the left side of $\tilde{A}^U$ and left side of $\tilde{B}^U$. $\overline{p}_8$ denotes the intersection points of the right side of $\tilde{A}^U$ and right side of $\tilde{B}^U$. $\overline{p}_9$ denotes the intersection points of the left side of $\tilde{A}^U$ and right side of $\tilde{B}^U$. $\overline{p}_{10}$ denotes the intersection points of the right side of $\tilde{A}^U$ and left side of $\tilde{B}^U$.

The coordinates of the ten possible points are summarized as follows.

$$\overline{p}_1:(\overline{x}_1, \overline{y}_1) = (a_2^U, 1)$$

$$\overline{p}_2:(\overline{x}_2, \overline{y}_2) = (a_3^U, 1)$$

$$\overline{p}_3:(\overline{x}_3, \overline{y}_3) = (b_2^U, 1)$$

$$\overline{p}_4:(\overline{x}_4, \overline{y}_4) = (b_3^U, 1)$$

$$\overline{p}_5:(\overline{x}_5, \overline{y}_5) = (\max(a_1^U, b_1^U), 0)$$

$$\overline{p}_6:(\overline{x}_6, \overline{y}_6) = (\min(a_4^U, b_4^U), 0)$$

$$\overline{p}_7:(\overline{x}_7, \overline{y}_7) = \left(\frac{b_1^U a_2^U - b_2^U a_1^U}{(a_2^U - a_1^U) - (b_2^U - b_1^U)}, \frac{b_1^U - a_1^U}{(a_2^U - a_1^U) - (b_2^U - b_1^U)}\right)$$

$$\overline{p}_8:(\overline{x}_8, \overline{y}_8) = \left(\frac{b_3^U a_4^U - b_4^U a_3^U}{(a_4^U - a_3^U) - (b_4^U - b_3^U)}, \frac{a_4^U - b_4^U}{(a_4^U - a_3^U) - (b_4^U - b_3^U)}\right)$$

$$\overline{p}_9:(\overline{x}_9, \overline{y}_9) = \left(\frac{b_4^U a_2^U - b_3^U a_1^U}{(a_2^U - a_1^U) - (b_3^U - b_4^U)}, \frac{b_4^U - a_1^U}{(a_2^U - a_1^U) - (b_3^U - b_4^U)}\right)$$

$$\overline{p}_{10}:(\overline{x}_{10}, \overline{y}_{10}) = \left(\frac{b_1^U a_3^U - b_2 a_4^U}{(a_3^U - a_4^U) - (b_2^U - b_1^U)}, \frac{b_1^U - a_4^U}{(a_3^U - a_4^U) - (b_2^U - b_1^U)}\right)$$

When two trapezoidal IT2 FSs $\tilde{A}$ and $\tilde{B}$ are given, the above ten points can be obtained. Then they will be checked through step *b)*, and the points passed check will be used to calculate the area of intersection polygon of two UMFs, i.e. $S_{\overline{A} \wedge \overline{B}}$.

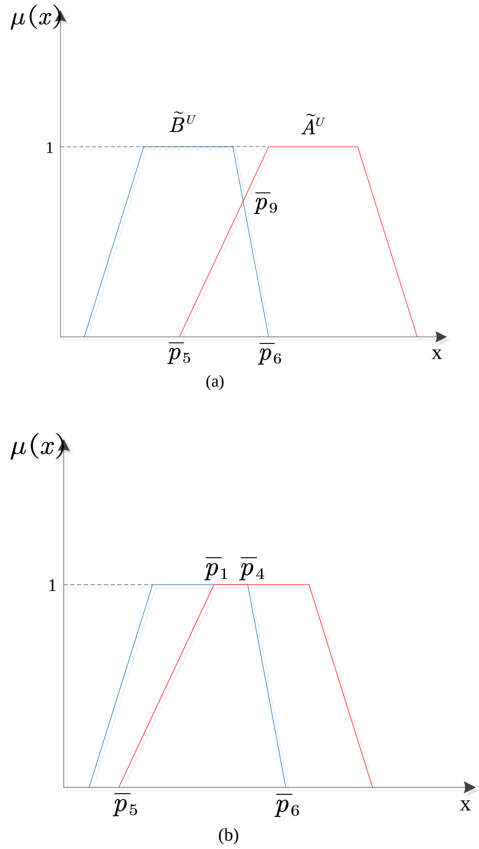

Fig. 3. Two examples of intersection situations of two UMFs, illustrating the three groups of points.

4) Ten possible points comprising the intersection polygon of two LMFs

For two LMFs $\tilde{A}^L$ with height h_A and $\tilde{B}^L$ with height h_B, there are also ten possible intersection points (Fig.4). Without loss of generality, it is supposed that $h_A < h_B$.

The ten possible intersection points can be divided into four groups. Two points located in x-axis do not need to check validity, which are denoted as $\underline{p}_1$ and $\underline{p}_2$. The left and right shoulder of $\tilde{A}^L$ and $\tilde{B}^L$ form four possible intersection points, denoted as $\underline{p}_3$ to $\underline{p}_6$. $\underline{p}_3$ denotes the intersection points of the left side of $\tilde{A}^L$ and left side of $\tilde{B}^L$. $\underline{p}_4$ denotes the intersection points of the right side of $\tilde{A}^L$ and right side of $\tilde{B}^L$. $\underline{p}_5$ denotes the intersection points of the left side of $\tilde{A}^L$ and right side of $\tilde{B}^L$. $\underline{p}_6$ denotes the intersection points of the right side of $\tilde{A}^L$ and left side of $\tilde{B}^L$.

Two shoulders of $\tilde{B}^L$, the higher LMF, may intersect with the top base of $\tilde{A}^L$, which form two possible points, denoted as $\underline{p}_7$ and $\underline{p}_8$, respectively. $\underline{p}_7$ denotes the point formed by the left shoulder of $\tilde{B}^L$ intersecting with the top base of $\tilde{A}^L$. $\underline{p}_8$ denotes the point formed by the right shoulder of $\tilde{B}^L$ intersecting with the top base of $\tilde{A}^L$.

There are two special points, denoted as $\underline{p}_9$ and $\underline{p}_{10}$, respectively. Though they are not intersection points, they are possible points composing the polygon (Fig.4. (d)). $\underline{p}_9$ is the left endpoint of the top base of $\tilde{A}^L$. $\underline{p}_{10}$ is the right endpoint of the top base of $\tilde{A}^L$. The criteria of checking these two points' validity is if $\underline{\mu}_{\tilde{B}}\left(\underline{x}_9\right)\left(\underline{\mu}_{\tilde{B}}\left(\underline{x}_{10}\right)\right) = h_B$.

The coordinates of the ten possible points that compose the intersection polygon of two LMFs are summarized as follows.

$$\underline{p}_1:\left(\underline{x}_1, \underline{y}_1\right) = \left(\max\left(a_1^L, b_1^L\right), 0\right)$$

$$\underline{p}_2:\left(\underline{x}_2, \underline{y}_2\right) = \left(\min\left(a_4^L, b_4^L\right), 0\right)$$

$$\underline{p}_3:\left(\underline{x}_3, \underline{y}_3\right) = \left(\frac{h_A \cdot a_1^L\left(b_2^L - b_1^L\right) - h_B \cdot b_1^L\left(a_2^L - a_1^L\right)}{h_A\left(b_2^L - b_1^L\right) - h_B\left(a_2^L - a_1^L\right)}, \frac{h_A h_B\left(a_1^L - b_1^L\right)}{h_A\left(b_2^L - b_1^L\right) - h_B\left(a_2^L - a_1^L\right)}\right)$$

$$\underline{p}_4:\left(\underline{x}_4, \underline{y}_4\right) = \left(\frac{h_A \cdot a_4^L\left(b_4^L - b_3^L\right) - h_B \cdot b_4^L\left(a_4^L - a_3^L\right)}{h_A\left(b_4^L - b_3^L\right) - h_B\left(a_4^L - a_3^L\right)}, \frac{h_A h_B\left(b_4^L - a_4^L\right)}{h_A\left(b_4^L - b_3^L\right) - h_B\left(a_4^L - a_3^L\right)}\right)$$

$$\underline{p}_5:\left(\underline{x}_5, \underline{y}_5\right) = \left(\frac{h_A \cdot a_1^L\left(b_4^L - b_3^L\right) + h_B \cdot b_4^L\left(a_2^L - a_1^L\right)}{h_A\left(b_4^L - b_3^L\right) + h_B\left(a_2^L - a_1^L\right)}, \frac{h_A h_B\left(b_4^L - a_1^L\right)}{h_A\left(b_4^L - b_3^L\right) + h_B\left(a_2^L - a_1^L\right)}\right)$$

$$\underline{p}_6:\left(\underline{x}_6, \underline{y}_6\right) = \left(\frac{h_A \cdot a_4^L\left(b_2^L - b_1^L\right) + h_B \cdot b_1^L\left(a_4^L - a_3^L\right)}{h_A\left(b_2^L - b_1^L\right) + h_B\left(a_4^L - a_3^L\right)}, \frac{h_A h_B\left(a_4^L - b_1^L\right)}{h_A\left(b_2^L - b_1^L\right) + h_B\left(a_4^L - a_3^L\right)}\right)$$

$$\underline{p}_7:\left(\underline{x}_7, \underline{y}_7\right) = \left(\frac{h_A}{h_B}\left(b_2^L - b_1^L\right) + b_1^L, h_A\right)$$

$$\underline{p}_8:\left(\underline{x}_8, \underline{y}_8\right) = \left(b_4^L - \frac{h_A}{h_B}\left(b_4^L - b_3^L\right),\ h_A\right)$$

$$\underline{p}_9:\left(\underline{x}_9, \underline{y}_9\right) = \left(a_2^L, h_A\right)$$

$$\underline{p}_{10}:\left(\underline{x}_{10}, \underline{y}_{10}\right) = \left(a_3^L, h_A\right)$$

When two trapezoidal IT2 FSs $\tilde{A}$ and $\tilde{B}$ are given, the above ten points can be obtained. Then they will be checked through step *b)*, and the points passed check will be used to calculate the area of intersection polygon of two LMFs, i.e. $S_{\underline{A} \wedge \underline{B}}$.

In summary, $S_{\bar{A} \wedge \bar{B}}$ and $S_{\underline{A} \wedge \underline{B}}$ are calculated separately. The processes of calculating these two elements are similar. First, obtain the ten possible intersection points according to the analysis above. Then check if these points are real intersection points. If they are real intersection points, they will be put into the set of points composing the intersection polygon. Finally, the area of the two intersection polygons are calculated via Eq. (12). The area is just what we want.

$S_{\bar{A}}, S_{\bar{B}}, S_{\underline{A}}, S_{\underline{B}}$ are already calculated via Eq.(8)-(11). After getting $S_{\bar{A} \wedge \bar{B}}$ and $S_{\underline{A} \wedge \underline{B}}$, the similarity of $\tilde{A}$ and $\tilde{B}$ can then be easily calculated according to Eq.(7).

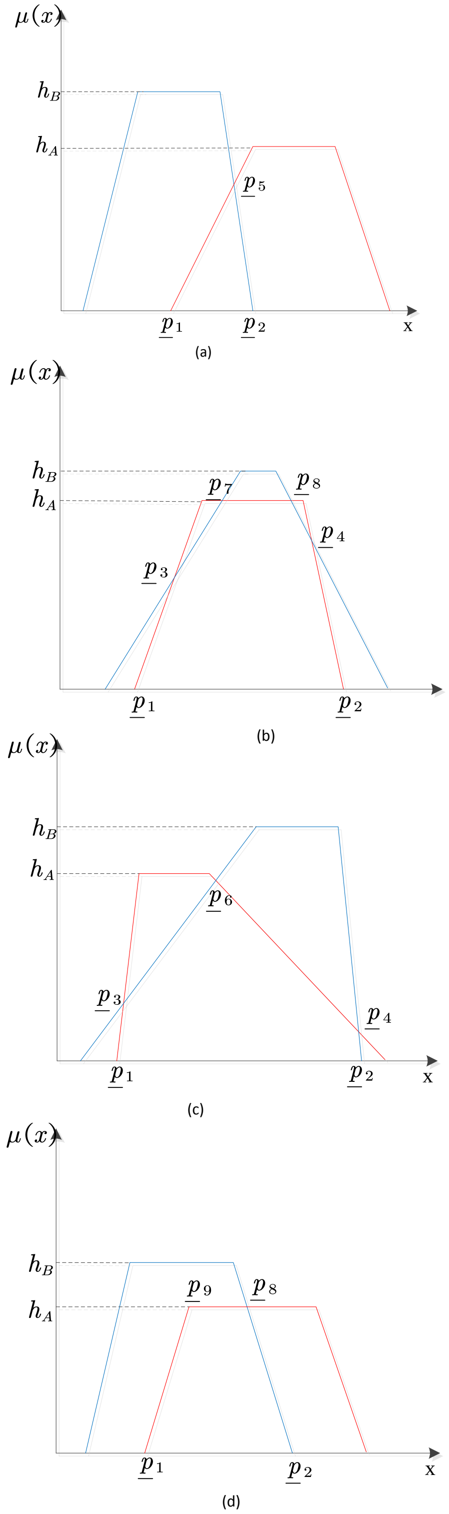

Fig. 4. Four examples of intersection situations of two LMFs, illustrating the four groups of points.

C. Numeric test

In this section, the area-based similarity measure proposed above is tested through a 32-IT2 FS dataset that includes 32 IT2 FSs with their associated words [28]. The 32 IT2 FSs represent 32 words from "None to very little" to "Maximum amount", from a small to a large meaning word. The data are shown in Table 1. The 32 trapezoidal IT2 FSs are ranked in an ascending order according to their average centroid. The similarity between any two of them is calculated using our analytical algorithm. The result is displayed in Table 8 (See Appendix).

From the similarity matrix (Table 8), we can draw the following conclusions:

1) The matrix is symmetric to its main diagonal. This is in accordance with the "symmetry" property of Jaccard similarity measure, i.e., $sm_J\left(\tilde{A},\tilde{B}\right)=sm_J\left(\tilde{B},\tilde{A}\right)$.

2) Observe a certain column or row, the data shows an ascending trend to 1 and then a descending trend. This is reasonable because the 32 IT2 FSs are listed from small to large. The similarity decreases as the distance increases.

3) Compare the result from our method with Wu's Jaccard measure [28], we can find that the value is almost same. Even if for the different element, the difference is no bigger than 0.01. This proves the validity of our method. But Wu's calculation involves integral and its result is an approximate solution. The result may change as different division of domain of discourse X. Our algorithm transforms the integral to area to get a steady analytical solution. This is why there is subtle difference between the results of the two algorithms.

IV. CWW EVALUATION MODEL BASED ON THE AREA SIMILARITY ALGORITHM

As Per-C is composed of three parts, i.e. encoder, CWW engine and decoder, when it is applied to evaluation problem, these three parts should be considered sequentially. The proposed model is composed of three stages, i.e., encoding, computing with words and decoding. Its flowchart is shown as Fig.5. In this section, how these procedures are implemented will be described in detail.

A. *Encoding*

The encoding part consists of three steps. The purpose of this encoding part is collecting linguistic evaluation from consumers and then transforming it to IT2 FS. After encoding, we obtain an evaluation matrix where all the evaluation information is represented by IT2 FS.

Step 1 Establish the codebook

In this step, we firstly decide the linguistic term set $\{s_0,s_1,s_2...,s_l\}$ that will be used by users to evaluate the performance of the evaluation object. Five or seven level linguistic term sets are mostly used. For example, we can define a five-level linguistic term set as {very bad, bad, average, good, very good}. Then, a group of target consumers are surveyed. They will be asked questions like "In a scale of 0-10, which interval do you think should be assigned to word "bad"?". For each word in the predefined linguistic term set, we get some intervals that people think can represent the word. Next, encoding methods [26, 27] will be used to construct the codebook. Finally, we get a codebook where each word is associated with an IT2 FS.

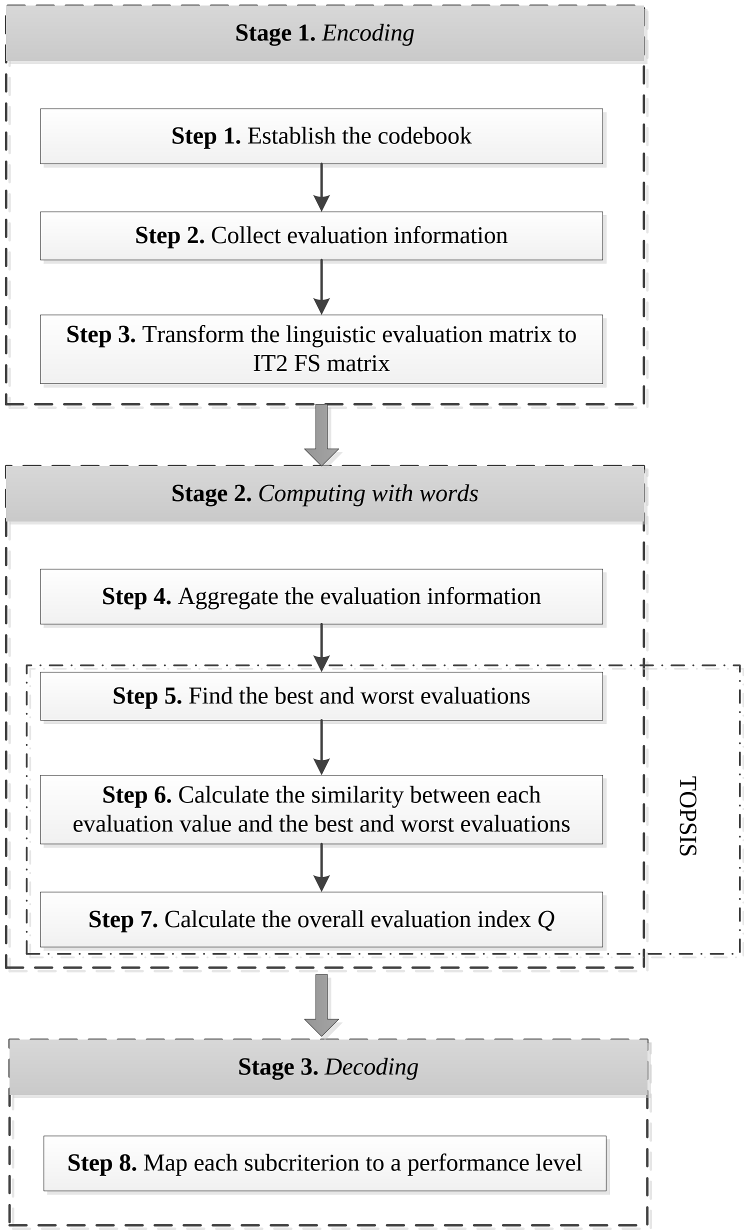

Fig. 5. Flowchart of the proposed CWW model for public transport service evaluation

Step 2 Collect evaluation information

For a certain service evaluation problem, the provided service is evaluated from n dimensions, which is called $D_1,D_2,...,D_n$. For dimension $D_j\,(j=1,2,...,n)$, it contains k_j subcriteria. Assume m people $(P_1,P_2,...,P_m)$ are investigated and they give their evaluation using words from linguistic term sets $\{s_0,s_1,s_2...,s_l\}$. After the investigation, we will get an evaluation matrix X containing the whole linguistic evaluation information, in which $x_{ijt}\,(i=1,2,...,m;\ j=1,2,...,n;\ \ t=1,2,...,k_j)$ represents the evaluation given by consumer P_i with respect to criterion c_{jt}.

$$X=\begin{array}{c} \\ \\ P_1 \\ P_2 \\ \vdots \\ P_m \end{array}\begin{array}{c} \begin{array}{ccccccccccccc} D_1 & & & & D_2 & & & & \cdots & D_n & & & \\ c_{11} & c_{12} & \cdots & c_{1k_1} & c_{21} & c_{22} & \cdots & c_{2k_2} & \cdots & c_{n1} & c_{n2} & \cdots & c_{nk_n} \end{array} \\ \left[\begin{array}{ccccccccccccc} x_{111} & x_{112} & \cdots & x_{11k_1} & x_{121} & x_{122} & \cdots & x_{12k_2} & \cdots & x_{1n1} & x_{1n2} & \cdots & x_{1nk_n} \\ x_{211} & x_{212} & \cdots & x_{21k_1} & x_{221} & x_{222} & \cdots & x_{22k_2} & \cdots & x_{2n1} & x_{2n2} & \cdots & x_{2nk_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots & \ddots & \vdots \\ x_{m11} & x_{m12} & \cdots & x_{m1k_1} & x_{m21} & x_{m22} & \cdots & x_{m2k_2} & \cdots & x_{mn1} & x_{mn2} & \cdots & x_{mnk_n} \end{array}\right] \end{array} \tag{13}$$

Step 3 Transform the linguistic evaluation matrix to IT2 FS matrix

In Step 1, we establish the associated relation between each word and an IT2 FS. In this step, each linguistic evaluation value x_{ijt} in X is transformed to its associated IT2 FS according to the relation. After that we obtain an IT2 FS evaluation matrix $\widetilde{X}$.

$$\widetilde{X}=\begin{array}{c} \\ \\ P_1 \\ P_2 \\ \vdots \\ P_m \end{array}\begin{array}{c} \begin{array}{ccccccccccccc} D_1 & & & & D_2 & & & & \cdots & D_n & & & \\ c_{11} & c_{12} & \cdots & c_{1k_1} & c_{21} & c_{22} & \cdots & c_{2k_2} & \cdots & c_{n1} & c_{n2} & \cdots & c_{nk_n} \end{array} \\ \left[\begin{array}{ccccccccccccc} \widetilde{x}_{111} & \widetilde{x}_{112} & \cdots & \widetilde{x}_{11k_1} & \widetilde{x}_{121} & \widetilde{x}_{122} & \cdots & \widetilde{x}_{12k_2} & \cdots & \widetilde{x}_{1n1} & \widetilde{x}_{1n2} & \cdots & \widetilde{x}_{1nk_n} \\ \widetilde{x}_{211} & \widetilde{x}_{212} & \cdots & \widetilde{x}_{21k_1} & \widetilde{x}_{221} & \widetilde{x}_{222} & \cdots & \widetilde{x}_{22k_2} & \cdots & \widetilde{x}_{2n1} & \widetilde{x}_{2n2} & \cdots & \widetilde{x}_{2nk_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots & \ddots & \vdots \\ \widetilde{x}_{m11} & \widetilde{x}_{m12} & \cdots & \widetilde{x}_{m1k_1} & \widetilde{x}_{m21} & \widetilde{x}_{m22} & \cdots & \widetilde{x}_{m2k_2} & \cdots & \widetilde{x}_{mn1} & \widetilde{x}_{mn2} & \cdots & \widetilde{x}_{mnk_n} \end{array}\right] \end{array} \tag{14}$$

B. *Computing with words*

This part consists of four steps. *Step 4* aggregates the collected evaluation to a comprehensive one-dimension matrix. *Step 5-7* incorporate the idea of TOPSIS to give each subcriterion an evaluation in the form of Q index.

Step 4 Aggregate the evaluation information

For the m IT2 FSs $\widetilde{x}_{ijt}(i=1,2,...,m)$ under each subcriterion c_{jt}, aggregate them via LWA (Eq.(15)) [19]. Because usually a large number of consumers are investigated, each consumer's weight is assumed equal to 1/m. The aggregated matrix $\hat{\widetilde{X}}$ is one-dimension, where each element $\widetilde{x}_{jt}$ is still an IT2 FS.

$$\widetilde{x}_{jt}=\sum_{i=1}^{m}\frac{\widetilde{x}_{ijt}}{m}\quad(j=1,2,...,n;\ t=1,2,...,k_j) \tag{15}$$

$$\hat{\widetilde{X}}=\begin{array}{c} \begin{array}{ccccccccccccc} D_1 & & & & D_2 & & & & \cdots & D_n & & & \\ c_{11} & c_{12} & \cdots & c_{1k_1} & c_{21} & c_{22} & \cdots & c_{2k_2} & \cdots & c_{n1} & c_{n2} & \cdots & c_{nk_n} \end{array} \\ \left[\begin{array}{ccccccccccccc} \widetilde{x}_{11} & \widetilde{x}_{12} & \cdots & \widetilde{x}_{1k_1} & \widetilde{x}_{21} & \widetilde{x}_{22} & \cdots & \widetilde{x}_{2k_2} & \cdots & \widetilde{x}_{n1} & \widetilde{x}_{n2} & \cdots & \widetilde{x}_{nk_n} \end{array}\right] \end{array} \tag{16}$$

Step 5 Find the best and worst evaluations

To do the CWW operation based on similarity of fuzzy sets, the idea of TOPSIS is introduced here. TOPSIS [30] is a classical multi-attribute decision making (MADM) method, whose main idea is that the best solution is the one which is closest to the positive ideal solution (PIS) and furthest to the negative ideal solution (NIS). Suppose there are total q $(q=k_1+k_2+...+k_n)$ subcriteria, they can be divided into two types: benefit criteria and cost criteria. Benefit criteria are the larger, the better, while cost criteria are the smaller, the better. The evaluation value of benefit criteria is kept what they are. For the cost criteria, the evaluation values should first be transform to their antonyms. Suppose an evaluation value is an IT2 FS $\widetilde{x}_{jt}$ and its antonyms FS is $\widetilde{x}'_{jt}$. Then their membership functions have the following relationship.

$$\mu_{\widetilde{x}'_{jt}}(x)=\mu_{\widetilde{x}_{jt}}(u-x)\ \ \forall x \tag{17}$$

where u is the upper limit of the domain of discourse of all FSs in the codebook.

The transformed evaluation matrix is $\widetilde{X}'$. Then compare q evaluation values $\widetilde{x}'_{jt}$ to find the best one and the worst one. Let $\widetilde{x}^+$ be the best evaluation and $\widetilde{x}^-$ be the worst evaluation. The best evaluation is the largest one and the worst evaluation is the smallest one. We can calculate the center of centroid [16] of each $\widetilde{x}_{jt}$ to determine $\widetilde{x}^+$ and $\widetilde{x}^-$.

Step 6 Calculate the similarity between each evaluation value and the best and worst evaluations

Calculate the similarity between each $\widetilde{x}'_{jt}$ and the two extreme evaluations, $\widetilde{x}^+$ and $\widetilde{x}^-$, determined in *Step 5*, respectively. The proposed analytical Jaccard similarity measure is employed here. The similarity between the jt_{th} IT2 FS $\widetilde{x}'_{jt}$ and the best evaluation $\widetilde{x}^+$ is denoted as s^+_{jt}. Similarly, the similarity between the jt_{th} IT2 FS $\widetilde{x}'_{jt}$ and the worst evaluation $\widetilde{x}^-$ is denoted as s^-_{jt}.

Step 7 Calculate the overall evaluation index

The overall evaluation index $Q_{jt}(0\leqslant Q_{jt}\leqslant 1)$ is a reflection of the quality of the evaluation, which considers both similarity to the best evaluation and the worst evaluation. The larger the Q_{jt}, the better the performance of the evaluated service quality is in the aspect c_{jt}. For subcriterion c_{jt}, Q_{jt} is calculated as Eq.(18). Then Q_{jt} is aggregated to overall evaluation index in the main dimension level. Assume the weight associated with subcriterion c_{jt} is w_{jt}, which is determined in advance. Let Q_j be the overall evaluation index of dimension D_j. Q_j is calculated through arithmetic weighted average (Eq.(19)).

$$Q_{jt}=\frac{s^-_{jt}}{s^-_{jt}+s^+_{jt}}\ ,\ j=1,2,...,n;\ t=1,2,...,k_j \tag{18}$$

$$\hat{Q}_j=\sum_{t=1}^{k_j}w_{jt}Q_{jt}\quad j=1,2,...,n \tag{19}$$

C. *Decoding*

In the decoding part, the Q indexes obtained in the last step are mapped to five performance levels. Finally, each subcriterion has a linguistic label indicating its performance.

Step 8 Map each subcriterion to a performance level

To better understand the performance of different dimensions, the q sub-criteria and n dimensions are mapped to five levels according to their Q value. The mapping rule is: 0-0.2: very bad evaluation, 0.2-0.4: bad evaluation, 0.4-0.6: regular evaluation, 0.6-0.8: good evaluation, 0.8-1: excellent evaluation.

After these three stages and eight steps, a CWW model for service evaluation is established. The original input is linguistic evaluations collected from the users, and the final output is also five linguistic levels, which can be easily understood by human. In this CWW application, one special thing is that the decoding process is partly accomplished the same time as computing with words process due to the incorporating of TOPSIS idea. When the similarity between each evaluation and the ideal evaluations are calculated in *Step 6*, the IT2 FSs are defuzzified to crisp number. In *Step 8*, we transform the crisp number evaluation to linguistic ratings.

V. CASE STUDY

In this section, the CWW evaluation model is applied to a public transport service evaluation problem to illustrate its effectiveness. Firstly, problem background is stated and the used data is presented. Then the three stages and eight steps are implemented to the public transport service evaluation problem. At last, the obtained results are compared to two evaluation methods, i.e., Type-1 (T1) fuzzy TOPSIS method and Type-2 (T2) defuzzification TOPSIS method.

A. *Problem description*

Public mass transit system is an effective way to relieve the current environmental and economic problems related to private transport. As a newly developed mass transit system in recent decades, bus rapid transit (BRT) system has gained prominence in many countries around the world. BRT is defined as a bus-based mass transit system aimed to provide fast and comfortable mobility at a lower cost. It combines the advantages of rail system and conventional bus system, as is called “surface subway”. BRT tries to obtain the high speed and reliability of metro system, at the same time reserves the flexibility and lower cost of bus system.

As a pioneer in establishing BRT system, Brazil built its first BRT line in Curitiba in 1974 and then expanded the successful experience to other 31 cities. Now a mature BRT system has been built in Brazil, which covers 871 km and delivers 12 million people. Therefore, it is a good choice to investigate the user satisfaction on BRT service based on the data collected in Brazil.

In this section, we show how to evaluate the service quality of a specific public transport form, i.e. BRT by CWW evaluation model. The data in [12] is adopted to illustrate our method. The evaluation was performed based on seven dimensions including the five dimensions in SERVQUAL [31] model and another two important dimensions appeared in the literature. These seven dimensions includes reliability, comfort, convenience, communication/information systems, technical security, accessibility and empathy. Each dimension is composed of some sub-criteria, for example, the first dimension “reliability” is composed of three sub-criteria reflecting the reliability of the BRT travel. All the criteria and sub-criteria are shown in Fig.6.

B. *Implementation of the proposed CWW model*

1) *Encoding*

The evaluation data were collected from 569 BRT passengers through online questionnaire [12]. Each respondent provides their evaluation using one of the five linguistic terms from “very dissatisfied” to “very satisfied”. Then the collected linguistic terms were transformed to triangular T1 FS according to predefined corresponding relation. Afterwards, the 569 triangular T1 FSs under each criterion were aggregated to one triangular T1 FS. The aggregated triangular T1 FSs are shown in the third column of Table 2.

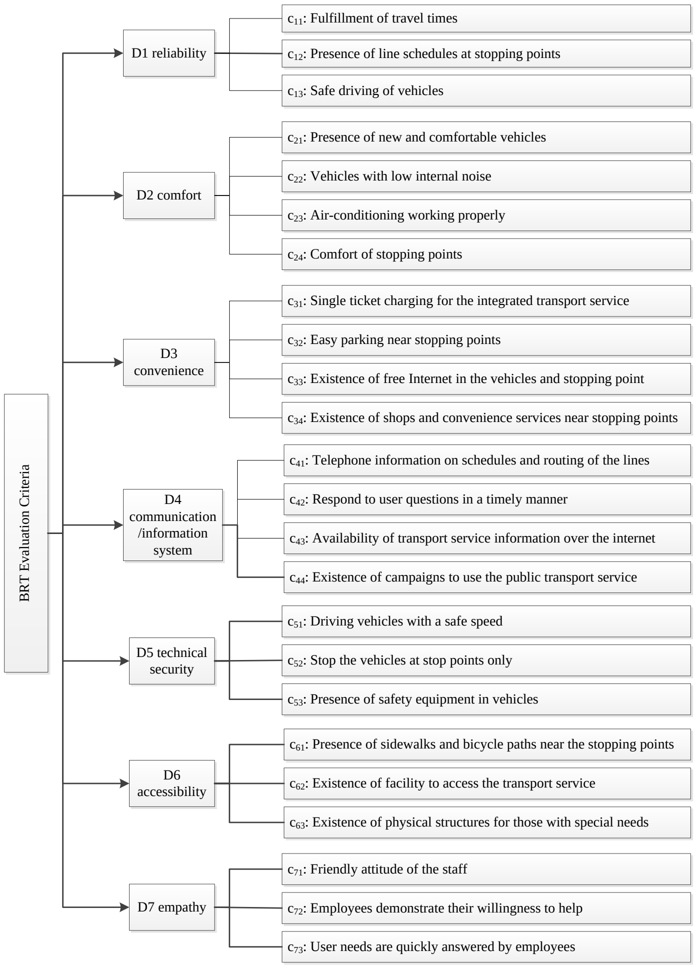

Fig. 6. Framework of BRT service evaluation criteria

As stated in the Introduction, IT2 FSs are preferred to model linguistic evaluation in the CWW process. But we cannot perform the investigation again to obtain the original data, so we choose to convert the aggregated triangular T1 FSs to trapezoidal IT2 FSs in the 32-codebook presented in Table 1. The centroid is a representation of a fuzzy set when all uncertainty disappears [28], therefore we can convert the triangular T1 FSs to IT2 FSs by comparing their centroid.

The membership function of a T1 FS F is denoted by Eq.(20). Firstly, the centroid of the triangular T1 FSs in Table 2 are calculated according to Eq.(21) and shown in the fourth column of Table 2. The center of centroid of the 32 IT2 FSs are already calculated via KM algorithm [32, 33], as shown in the last column of Table1. Then the triangular T1 FSs are converted one by one. The converting rule is to find the IT2 FS from the 32 IT2 FSs whose center of centroid is closest to the centroid of a triangular T1 FS. The final converting result is shown in the last two columns of Table 2. By the converting, we get the evaluation represented by IT2 FSs and thereupon the encoding stage is accomplished.

After the encoding, every sub-criterion now has an IT2 FS evaluation. From Table 2, we can find the IT2 FSs vary from “Somewhat small” to “Considerable Amount”, including 11 words. Their similarity between each other is distracted from Table 8 and put in Table 3.

$$\mu_F(x) = \begin{cases} \dfrac{x-a}{b-a} & a < x \leq b \\ \dfrac{x-c}{b-c} & b \leq x < c \\ 0 & otherwise \end{cases} \quad (20)$$

$$\begin{aligned} c(F) &= \frac{\int_a^c x\mu(x)dx}{\int_a^c \mu(x)dx} \\ &= \frac{\int_a^b x \cdot \frac{x-a}{b-a}dx + \int_b^c x \cdot \frac{x-c}{b-c}dx}{\int_a^b \frac{x-a}{b-a}dx + \int_b^c \frac{x-c}{b-c}dx} \\ &= \frac{\frac{a^3+2b^3-3ab^2}{3(b-a)} - \frac{c^3+2b^3-3b^2c}{3(b-c)}}{c-a} \end{aligned} \quad (21)$$

2) Computing with words

In this problem, the best evaluation is obviously "Considerable Amount", and the worst evaluation is "Somewhat small". The area similarity between each evaluation of each sub-criterion and the two ideal evaluations are calculated and shown in the third and fourth columns of Table 4, respectively. Then Q_{jt} are calculated and listed in the fifth column of Table 4. The of the twenty-four sub-criteria are aggregated to the seven main criteria level. Because the weight information of each sub-criterion is not known, we assume they are of same importance and aggregated by Eq.(19). The results are shown in the second and third columns of Table 5.

3) Decoding

The 24 sub-criteria are mapped to five levels according to their Q_{jt}. The mapping results are shown in the last column of Table 4.

Observe Table 4, we can find that c_{12}, i.e. presence of line schedules at stopping points, performs worst. This indicates that the present BRT system lacks of this equipment and there is an intensive need for it. There are eight sub-criteria got an evaluation "bad", which concentrate on the dimension "convenience" and "communication/information system". Therefore, the operators should focus on these two aspects and seek improvements. One criterion is assessed as "regular" and five sub-criteria are evaluated as "good", which means a satisfying performance. Nine sub-criteria are thought to be "excellent", mainly appearing in the three dimensions: "comfort", "technical security" and "empathy". Overall, most aspects get "bad" or "excellent" label, which proves that our CWW model can give a clear and distinguished evaluation.

Now we move to a higher level to analyze the performance of the BRT system service. The performance of the BRT system in the seven dimension level is shown in the red line of Fig.7. Observe Table 5 and Fig.7, dimension "communication/information system" gets a "bad" evaluation and the other criteria get either "regular" or "good". This indicates that the communication and information system, including communication with passengers in telephone or internet form, needs more attention. As a whole, the BRT system's performance is satisfying, but there is still large space to improve. When the Q_{jt} for the 24 subcriteria are aggregated, the performance of them counteract with each other. Therefore, when a dimension contains bad evaluation and excellent evaluation at the same time in subcriteria level, it may become regular in dimension level. This indicates that the subcriteria level contains more information. BRT operators should focus more on microcosmic level and find the weaker respects to take specified measures.

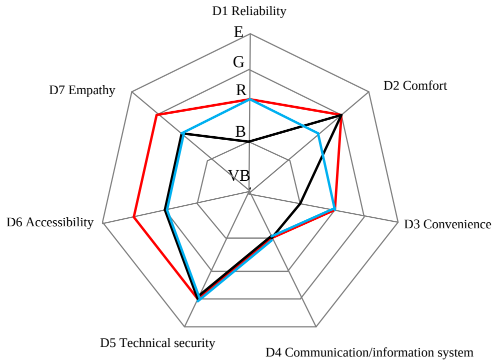

Fig. 7. Radar chart for classification results of the three methods on dimension level. Red line shows the results of the CWW model in this paper. Blue line shows the results of T1 fuzzy TOPSIS method. Black line shows the results of T2 defuzzification TOPSIS method.

C. Comparative analysis

In this subsection, the results obtained in this paper will be compared with T1 fuzzy TOPSIS method [12] and T2 defuzzification TOPSIS method [25]. The comparisons will be done in the 24 subcriteria level and 7 main dimensions level, respectively. The results will be contrasted first and the consistency or difference will be analyzed then.

1) Comparison with T1 fuzzy TOPSIS method [12]

a) 24-Subcriteria level

The TOPSIS method is introduced to measure the performance of the BRT system in [12] as well and the Q index, which is called C instead, is also obtained. Using the mapping rule defined in *Step 8* (Section III), the 24 subcriteria can also be classified to five levels according to the C index. Thus, we can compare the results gained using our CWW model and the T1 fuzzy TOPSIS method. The classification results of the two methods are displayed in Fig.8 and Table 6. Observe Fig.8 and Table 6, we can find that in our results, the subcriteria distributed more dispersedly, whereas in T1 fuzzy TOPSIS method, more subcriteria are closer to the central class "regular". Note that there is no subcriterion in "very bad" class and only one subcriteria "c_{52}" is classified as "excellent" in T1 fuzzy TOPSIS method, while nine subcriteria are sorted to "excellent" class in our model. The difference shows that our model can better distinguished the subcriteria, which is helpful in aiding operators of BRT system to find the better performed and weaker aspects. The reason why our CWW model has a better distinguishing performance may be that IT2 FSs are used to model linguistic evaluations in the CWW process while T1 FSs are used in T1 fuzzy TOPSIS method. Compared with T1 FS, IT2 FS can capture more uncertainties of words and hence propagate to the final results.

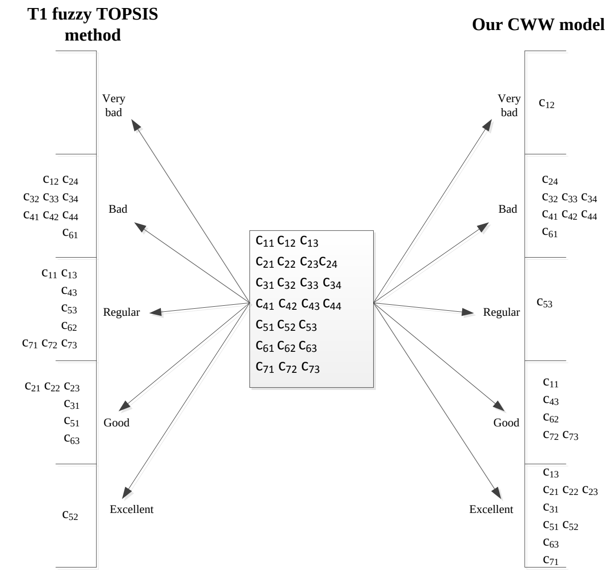

Fig. 8. The classifying results comparison of two methods.

b) Seven main dimensions level

The C_j index and corresponding level of the seven dimensions are listed in the fourth and fifth row of Table 5, respectively. Comparing the results obtained in our CWW model and T1 fuzzy TOPSIS method, there is no major difference between the two approaches. Four dimensions, i.e. "D1, D3, D4 and D5", are classified to the same classes. For the three inconsistent dimensions, i.e. "D2, D6 and D7", our method classifies them into "good", while T1 fuzzy TOPSIS method sorts them into "regular". Overall, the difference is not significant. Another finding is that the results of the seven main dimensions give less information than that of the 24 subcriteria. Dimensions are all classified to "regular" or "good" except for "D4" getting a "bad" label. This can only tell us the integral BRT service is fair. To better understand which aspects are weaker, analysis from the microcosmic perspective, i.e. the 24 subcriteria level, is more meaningful.

2) Comparison with defuzzification TOPSIS method [25]

The defuzzification TOPSIS method firstly converted IT2 FSs to crisp numbers, then employed standard TOPSIS steps. When we use this method to solve the public transport service evaluation problem, the sorting results for each criterion are shown in the fourth column of Table 6. The sorting results for each dimension are represented in black line in Fig.7.

From Table 6 we can see that the sorting results are completely same between T2 defuzzification TOPSIS method and our CWW model in the "very bad" and "bad" level. However, the other three levels "regular", "good", "excellent" are different. The T2 defuzzification TOPSIS method sorted many criteria to "regular" level, while in our CWW model most of them are sorted to "good" or "excellent". Sorting a criterion to "regular" does not offer any useful information to operators because operators cannot make a judgement about its performance with a label "regular". Therefore, our CWW model can provides more effective information to decision makes by give a more separate classification result.

The reason of difference between two methods lies on the processing on IT2 FSs that are used to model evaluations both in these two methods. The T2 defuzzification TOPSIS method's defuzzifies IT2 FSs to crisp numbers, which leads to a large amount of information lost. Compared with the defuzzification, our area similarity measure directly calculates the similarity between two IT2 FSs, which reserves information and uncertainty as much as possible.

VI. Conclusions

Most existing service evaluation methods transform consumers' assessment to crisp number or T1 FS to perform calculation process. Considering the advantage of CWW in dealing with problems involved with humans' evaluation, we have designed a CWW evaluation model in this paper. First, consumers' evaluation is modeled by IT2 FS to reserve its imprecision, subjectivity and uncertainty. Then, the idea of TOPSIS is introduced as CWW engine. The area similarity measure directly calculate similarity between IT2 FSs, which overcomes the information loss of defuzzification. Finally, the CWW model gives each criterion a linguistic label as the evaluation output.

Comparative analysis with two existing evaluation methods shows our CWW model can output more scattered sorts, therefore can offer more information to decision-makers. Though the CWW model is applied to a specific service evaluation problem in this paper, it can be applied to other evaluation and decision-making problems as well.

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APPENDIX

TABLE I. THE 32-IT2 FSS DATASET

Word	**IT2 FS**	**Center of centroid** $c(\tilde{A}_i)$
1. None to very little	[(0, 0, 0.14, 1.97), (0, 0, 0.05, 0.66, 1)]	0.47
2. Teeny-weeny	[(0, 0, 0.14, 1.97), (0, 0, 0.05, 0.66, 1)]	0.56
3. A smidgen	[(0, 0, 0.26, 2.63), (0, 0, 0.05, 0.63, 1)]	0.63
4. Tiny	[(0, 0, 0.36, 2.63), (0, 0, 0.05, 0.63, 1)]	0.64
5. Very small	[(0, 0, 0.64, 2.47), (0, 0, 0.10, 1.16, 1)]	0.66
6. Very little	[(0, 0, 0.64, 2.63), (0, 0, 0.09, 0.99, 1)]	0.67
7. A bit	[(0.59, 1.50, 2.00, 3.41), (0.79, 1.68, 1.68, 2.21, 0.74)]	1.75
8. Little	[(0.38, 1.50, 2.50, 4.62), (1.09, 1.83, 1.83, 2.21, 0.53)]	2.13
9. Low amount	[(0.09, 1.25, 2.50, 4.62), (1.67, 1.92, 1.92, 2.21, 0.30)]	2.19
10. Small	[(0.09, 1.50, 3.00, 4.62), (1.79, 2.28, 2.28, 2.81, 0.40)]	2.32
11. Somewhat small	[(0.59, 2.00, 3.25, 4.41), (2.29, 2.70, 2.70, 3.21, 0.42)]	2.59
12. Some	[(0.38, 2.50, 5.00, 7.83), (2.88, 3.61, 3.61, 4.21, 0.35)]	3.90
13. Some to moderate	[(1.17, 3.50, 5.50, 7.83), (4.09, 4.65, 4.65, 5.41, 0.40)]	4.56
14. Moderate amount	[(2.59, 4.00, 5.50, 7.62), (4.29, 4.75, 4.75, 5.21, 0.38)]	4.95
15. Fair amount	[(2.17, 4.25, 6.00, 7.83), (4.79, 5.29, 5.29, 6.02, 0.41)]	5.13
16. Medium	[(3.59, 4.75, 5.50, 6.91), (4.86, 5.03, 5.03, 5.14, 0.27)]	5.19
17. Modest amount	[(3.59, 4.75, 6.00, 7.41), (4.79, 5.30, 5.30, 5.71, 0.42)]	5.41
18. Good amount	[(3.38, 5.50, 7.50, 9.62), (5.79, 6.50, 6.50, 7.21, 0.41)]	6.50
19. Sizeable	[(4.38, 6.50, 8.00, 9.41), (6.79, 7.38, 7.38, 8.21, 0.49)]	7.16
20. Quite a bit	[(4.38, 6.50, 8.00, 9.41), (6.79, 7.38, 7.38, 8.21, 0.49)]	7.16
21. Considerable amount	[(4.38, 6.50, 8.25, 9.62), (7.19, 7.58, 7.58, 8.21, 0.37)]	7.25
22. Substantial amount	[(5.38, 7.50, 8.75, 9.81), (7.79, 8.22, 8.22, 8.81, 0.45)]	7.90
23. A lot	[(5.38, 7.50, 8.75, 9.83), (7.69, 8.19, 8.19, 8.81, 0.47)]	7.91
24. High amount	[(5.38, 7.50, 8.75, 9.81), (7.79, 8.30, 8.30, 9.21, 0.53)]	8.01
25. Very sizeable	[(5.38, 7.50, 9.00, 9.81), (8.29, 8.56, 8.56, 9.21, 0.38)]	8.03
26. Large	[(5.98, 7.75, 8.60, 9.52), (8.03, 8.36, 8.36, 9.17, 0.57)]	8.12
27. Very large	[(7.37, 9.41, 10, 10), (8.72, 9.91, 10, 10, 1)]	9.30
28. Humongous amount	[(7.37, 9.82, 10, 10), (9.74, 9.98, 10, 10, 1)]	9.31
29. Huge amount	[(7.37, 9.59, 10, 10), (8.95, 9.93, 10, 10, 1)]	9.34
30. Very high amount	[(7.37, 9.73, 10, 10), (9.34, 9.95, 10, 10, 1)]	9.37
31. Extreme amount	[(7.37, 9.82, 10, 10), (9.37, 9.95, 10, 10, 1)]	9.38
32. Maximum amount	[(8.68, 9.91, 10, 10), (9.61, 9.97, 10, 10, 1)]	9.69

TABLE II. AGGREGATED TRIANGULAR T1 FSS FOR EACH SUB-CRITERION AND THEIR CORRESPONDING IT2 FSS

Criterion	Sub-criterion	Triangular T1 FSs	centroid	Corresponding IT2 FS	Corresponding word
D1 reliability	c11	(3.0237, 4.8343, 6.7396)	4.87	[(2.59, 4.00, 5.50, 7.62), (4.29, 4.75, 4.75, 5.21, 0.38)]	Moderate amount
	c12	(1.3077, 2.7870, 4.7574)	2.95	[(0.59, 2.00, 3.25, 4.41), (2.29, 2.70, 2.70, 3.21, 0.42)]	Somewhat small
	c13	(3.8698, 5.7574, 7.5740)	5.73	[(3.59, 4.75, 6.00, 7.41), (4.79, 5.30, 5.30, 5.71, 0.42)]	Modest amount
D2 comfort	c21	(4.4615, 6.3846, 8.0710)	6.31	[(3.38, 5.50, 7.50, 9.62), (5.79, 6.50, 6.50, 7.21, 0.41)]	Good Amount
	c22	(4.2840, 6.1834, 7.8994)	6.12	[(3.38, 5.50, 7.50, 9.62), (5.79, 6.50, 6.50, 7.21, 0.41)]	Good Amount
	c23	(3.9172, 5.8047, 7.5680)	5.76	[(3.59, 4.75, 6.00, 7.41), (4.79, 5.30, 5.30, 5.71, 0.42)]	Modest amount
	c24	(2.2249, 3.8757, 5.8107)	3.97	[(0.38, 2.50, 5.00, 7.83), (2.88, 3.61, 3.61, 4.21, 0.35)]	Some
D3 convenience	c31	(4.7101, 6.5976, 8.1361)	6.48	[(3.38, 5.50, 7.50, 9.62), (5.79, 6.50, 6.50, 7.21, 0.41)]	Good Amount
	c32	(2.0118, 3.6036, 5.5385)	3.72	[(0.38, 2.50, 5.00, 7.83), (2.88, 3.61, 3.61, 4.21, 0.35)]	Some
	c33	(2.1420, 3.8757, 5.8107)	3.94	[(0.38, 2.50, 5.00, 7.83), (2.88, 3.61, 3.61, 4.21, 0.35)]	Some
	c34	(1.5621, 3.1420, 5.1065)	3.27	[(0.38, 2.50, 5.00, 7.83), (2.88, 3.61, 3.61, 4.21, 0.35)]	Some
D4 communication/ information system	c41	(1.9527, 3.5444, 5.4793)	3.66	[(0.38, 2.50, 5.00, 7.83), (2.88, 3.61, 3.61, 4.21, 0.35)]	Some
	c42	(1.9349, 3.6509, 5.6331)	3.74	[(0.38, 2.50, 5.00, 7.83), (2.88, 3.61, 3.61, 4.21, 0.35)]	Some
	c43	(3.0651, 4.9053, 6.7751)	4.92	[(2.59, 4.00, 5.50, 7.62), (4.29, 4.75, 4.75, 5.21, 0.38)]	Moderate amount
	c44	(1.8935, 3.4970, 5.4438)	3.61	[(0.38, 2.50, 5.00, 7.83), (2.88, 3.61, 3.61, 4.21, 0.35)]	Some
D5 technical security	c51	(4.4260, 6.3491, 8.0828)	6.29	[(3.38, 5.50, 7.50, 9.62), (5.79, 6.50, 6.50, 7.21, 0.41)]	Good Amount
	c52	(5.6213, 7.5799, 9.0059)	7.40	[(4.38, 6.50, 8.25, 9.62), (7.19, 7.58, 7.58, 8.21, 0.37)]	Considerable Amount
	c53	(2.8107, 4.6568, 6.5562)	4.67	[(1.17, 3.50, 5.50, 7.83), (4.09, 4.65, 4.65, 5.41, 0.40)]	Some to moderate
D6 accessibility	c61	(2.2308, 3.9704, 5.8994)	4.03	[(0.38, 2.50, 5.00, 7.83), (2.88, 3.61, 3.61, 4.21, 0.35)]	Some
	c62	(3.1598, 4.9645, 6.8225)	4.98	[(2.59, 4.00, 5.50, 7.62), (4.29, 4.75, 4.75, 5.21, 0.38)]	Moderate amount
	c63	(3.9763, 5.8757, 7.6272)	5.83	[(3.59, 4.75, 6.00, 7.41), (4.79, 5.30, 5.30, 5.71, 0.42)]	Modest amount
D7 empathy	c71	(3.3669, 5.2959, 7.2130)	5.29	[(3.59, 4.75, 5.50, 6.91), (4.86, 5.03, 5.03, 5.14, 0.27)]	Medium
	c72	(3.1065, 5.0000, 6.9053)	5.00	[(2.59, 4.00, 5.50, 7.62), (4.29, 4.75, 4.75, 5.21, 0.38)]	Moderate amount
	c73	(2.8994, 4.7633, 6.6805)	4.86	[(2.59, 4.00, 5.50, 7.62), (4.29, 4.75, 4.75, 5.21, 0.38)]	Moderate amount

TABLE III. SIMILARITY BETWEEN THE 11 WORDS FROM "SOMEWHAT SMALL" TO "CONSIDERABLE AMOUNT"

	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	21.
11. Somewhat small	1	0.4272	0.2585	0.1167	0.1310	0.0311	0.0273	0.0232	0	0	0
12. Some	0.4272	1	0.7138	0.5557	0.5378	0.3650	0.3794	0.2625	0.1578	0.1578	0.1564
13. Some to moderate	0.2585	0.7138	1	0.7479	0.6985	0.4504	0.5062	0.3282	0.1947	0.1947	0.1927
14. Moderate amount	0.1167	0.5557	0.7479	1	0.7857	0.5991	0.6349	0.3699	0.2129	0.2129	0.2103
15. Fair amount	0.1310	0.5378	0.6985	0.7857	1	0.5222	0.6891	0.4218	0.2485	0.2485	0.2456
16. Medium	0.0311	0.3650	0.4504	0.5991	0.5222	1	0.7570	0.3674	0.1897	0.1897	0.1869

17. Modest amount	0.0273	0.3794	0.5062	0.6349	0.6891	0.7570	1	0.4578	0.2580	0.2580	0.2544
18. Good amount	0.0232	0.2625	0.3282	0.3699	0.4218	0.3674	0.4578	1	0.6359	0.6359	0.6272
19. Sizeable	0	0.1578	0.1947	0.2129	0.2485	0.1897	0.2580	0.6359	1	1	0.8987
20. Quite a bit	0	0.1578	0.1947	0.2129	0.2485	0.1897	0.2580	0.6359	1	1	0.8987
21. Considerable amount	0	0.1564	0.1927	0.2103	0.2456	0.1869	0.2544	0.6272	0.8987	0.8987	1

TABLE IV. THE SIMILARITY BETWEEN THE 24 SUBCRITERIA AND IDEAL EVALUATIONS AND THE Q INDEX OF EACH SUBCRITERIA

Criterion	Sub-criterion	s_{jt}^{+}	s_{jt}^{-}	Q_{jt}	level
D1 reliability	c11	0.1167	0.2103	0.6431	Good
	c12	1.0000	0.0000	0.0000	Very bad
	c13	0.0273	0.2544	0.9031	Excellent
D2 comfort	c21	0.0232	0.6272	0.9643	Excellent
	c22	0.0232	0.6272	0.9643	Excellent
	c23	0.0273	0.2544	0.9031	Excellent
	c24	0.4272	0.1564	0.2680	Bad
D3 convenience	c31	0.0232	0.6272	0.9643	Excellent
	c32	0.4272	0.1564	0.2680	Bad
	c33	0.4272	0.1564	0.2680	Bad
	c34	0.4272	0.1564	0.2680	Bad
D4 communication/information system	c41	0.4272	0.1564	0.2680	Bad
	c42	0.4272	0.1564	0.2680	Bad
	c43	0.1167	0.2103	0.6431	Good
	c44	0.4272	0.1564	0.2680	Bad
D5 technical security	c51	0.0232	0.6272	0.9643	Excellent
	c52	0.0000	1.0000	1.0000	Excellent
	c53	0.2585	0.1927	0.4271	Regular
D6 accessibility	c61	0.4272	0.1564	0.2680	Bad
	c62	0.1167	0.2103	0.6431	Good
	c63	0.0273	0.2544	0.9031	Excellent
D7 empathy	c71	0.0311	0.1869	0.8573	Excellent
	c72	0.1167	0.2103	0.6431	Good
	c73	0.1167	0.2103	0.6431	Good

TABLE V. THE PERFORMANCE OF THE FIVE MAIN CRITERIA

Criterion	Q_j	level with our CWW model	C_j	level with T1 fuzzy TOPSIS method [12]	level with T2 defuzzification TOPSIS method [25]
D1 reliability	0.5154	Regular	0.4343	Regular	Bad
D2 comfort	0.7749	good	0.5731	Regular	Good
D3 convenience	0.4421	Regular	0.4123	Regular	Bad
D4communication/ information system	0.3618	Bad	0.3619	Bad	Bad
D5 technical security	0.7971	Good	0.6520	Good	Good
D6 accessibility	0.6047	Good	0.4928	Regular	Regular
D7 empathy	0.7145	Good	0.5034	Regular	Regular

TABLE VI. FIVE CLASSES AND THE SUBCRITERIA EACH CLASS CONTAINING USING THE THREE METHODS

Label	CWW model	T1 fuzzy TOPSIS method [12]	T2 defuzzification TOPSIS method [25]
Very bad	c_{12}		c_{12}
Bad	c_{24} c_{32} c_{33} c_{34} c_{41} c_{42} c_{44} c_{61}	c_{12} c_{24} c_{32} c_{33} c_{34} c_{41} c_{42} c_{44} c_{61}	c_{24} c_{32} c_{33} c_{34} c_{41} c_{42} c_{44} c_{61}
Regular	c_{53}	c_{11} c_{13} c_{43} c_{53} c_{62} c_{71} c_{72} c_{73}	c_{11} c_{13} c_{23} c_{43} c_{53} c_{62} c_{63} c_{71} c_{72} c_{73}
Good	c_{11} c_{43} c_{62} c_{72} c_{73}	c_{21} c_{22} c_{23} c_{31} c_{51} c_{63}	
Excellent	c_{13} c_{21} c_{22} c_{23} c_{31} c_{51} c_{52} c_{63} c_{71}	c_{52}	c_{21} c_{22} c_{31} c_{51} c_{52}

TABLE VII. THE SIMILARITY BETWEEN ANY TWO FUZZY SETS IN THE 32-IT2 FS DATABASE USING THE PROPOSED ANALYTICAL ALGORITHM

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1	1	0.80	0.78	0.75	0.65	0.65	0.11	0.11	0.16	0.13	0.08	0.05	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0.80	1	0.63	0.61	0.51	0.52	0.12	0.12	0.17	0.14	0.08	0.05	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0.78	0.63	1	0.97	0.80	0.82	0.19	0.18	0.24	0.21	0.14	0.09	0.04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0.75	0.61	0.97	1	0.82	0.84	0.19	0.19	0.24	0.21	0.14	0.09	0.04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0.65	0.51	0.80	0.82	1	0.92	0.18	0.17	0.23	0.19	0.12	0.08	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0.65	0.52	0.82	0.84	0.92	1	0.20	0.19	0.25	0.21	0.14	0.09	0.04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0.11	0.12	0.19	0.19	0.18	0.20	1	0.62	0.51	0.46	0.40	0.21	0.11	0.02	0.04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0.11	0.12	0.18	0.19	0.17	0.19	0.62	1	0.85	0.77	0.66	0.35	0.22	0.10	0.12	0.03	0.03	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0.16	0.17	0.24	0.24	0.23	0.25	0.51	0.85	1	0.83	0.65	0.35	0.21	0.10	0.12	0.03	0.03	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0.13	0.14	0.21	0.21	0.19	0.21	0.46	0.77	0.83	1	0.74	0.39	0.24	0.11	0.13	0.04	0.03	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0.08	0.08	0.14	0.14	0.12	0.14	0.40	0.66	0.65	0.74	1	0.43	0.26	0.12	0.13	0.03	0.03	0.02	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0.05	0.05	0.09	0.09	0.08	0.09	0.21	0.35	0.35	0.39	0.43	1	0.71	0.56	0.54	0.37	0.38	0.26	0.16	0.16	0.16	0.08	0.08	0.08	0.08	0.05	0	0	0	0	0	0
13	0.01	0.01	0.04	0.04	0.03	0.04	0.11	0.22	0.21	0.24	0.26	0.71	1	0.75	0.70	0.45	0.51	0.33	0.19	0.19	0.19	0.10	0.10	0.09	0.10	0.06	0	0	0	0	0	0
14	0	0	0	0	0	0	0.02	0.10	0.10	0.11	0.12	0.56	0.75	1	0.79	0.60	0.63	0.37	0.21	0.21	0.21	0.10	0.10	0.10	0.10	0.06	0	0	0	0	0	0
15	0	0	0	0	0	0	0.04	0.12	0.12	0.13	0.13	0.54	0.70	0.79	1	0.52	0.69	0.42	0.25	0.25	0.25	0.12	0.12	0.12	0.12	0.08	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0.03	0.03	0.04	0.03	0.37	0.45	0.60	0.52	1	0.76	0.37	0.19	0.19	0.19	0.07	0.07	0.07	0.07	0.03	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0.03	0.03	0.03	0.03	0.38	0.51	0.63	0.69	0.76	1	0.46	0.26	0.26	0.25	0.11	0.11	0.11	0.11	0.07	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0.03	0.03	0.03	0.02	0.26	0.33	0.37	0.42	0.37	0.46	1	0.64	0.64	0.63	0.40	0.39	0.38	0.39	0.32	0.10	0.10	0.10	0.10	0.10	0.03
19	0	0	0	0	0	0	0	0	0	0	0	0.16	0.19	0.21	0.25	0.19	0.26	0.64	1	1	0.90	0.52	0.52	0.51	0.50	0.43	0.11	0.12	0.11	0.11	0.11	0.02
20	0	0	0	0	0	0	0	0	0	0	0	0.16	0.19	0.21	0.25	0.19	0.26	0.64	1	1	0.90	0.52	0.52	0.51	0.50	0.43	0.11	0.12	0.11	0.11	0.11	0.02
21	0	0	0	0	0	0	0	0	0	0	0	0.16	0.19	0.21	0.25	0.19	0.25	0.63	0.90	0.90	1	0.60	0.60	0.58	0.58	0.50	0.14	0.15	0.14	0.14	0.14	0.04
22	0	0	0	0	0	0	0	0	0	0	0	0.08	0.10	0.10	0.12	0.07	0.11	0.40	0.52	0.52	0.60	1	0.99	0.95	0.88	0.73	0.22	0.23	0.21	0.22	0.21	0.08
23	0	0	0	0	0	0	0	0	0	0	0	0.08	0.10	0.10	0.12	0.07	0.11	0.39	0.52	0.52	0.60	0.99	1	0.94	0.87	0.72	0.22	0.23	0.21	0.22	0.21	0.08
24	0	0	0	0	0	0	0	0	0	0	0	0.08	0.09	0.10	0.12	0.07	0.11	0.38	0.51	0.51	0.58	0.95	0.94	1	0.90	0.77	0.22	0.22	0.21	0.21	0.21	0.07
25	0	0	0	0	0	0	0	0	0	0	0	0.08	0.10	0.10	0.12	0.07	0.11	0.39	0.50	0.50	0.58	0.88	0.87	0.90	1	0.72	0.25	0.24	0.24	0.23	0.23	0.08
26	0	0	0	0	0	0	0	0	0	0	0	0.05	0.06	0.06	0.08	0.03	0.07	0.32	0.43	0.43	0.50	0.73	0.72	0.77	0.72	1	0.21	0.20	0.19	0.19	0.19	0.05
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.10	0.11	0.11	0.14	0.22	0.22	0.22	0.25	0.21	1	0.67	0.91	0.79	0.76	0.40
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.10	0.12	0.12	0.15	0.23	0.23	0.22	0.24	0.20	0.67	1	0.74	0.86	0.89	0.52
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.10	0.11	0.11	0.14	0.21	0.21	0.21	0.24	0.19	0.91	0.74	1	0.87	0.84	0.44
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.10	0.11	0.11	0.14	0.22	0.22	0.21	0.23	0.19	0.79	0.86	0.87	1	0.97	0.51
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.10	0.11	0.11	0.14	0.21	0.21	0.21	0.23	0.19	0.76	0.89	0.84	0.97	1	0.52
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.03	0.02	0.02	0.04	0.08	0.08	0.07	0.08	0.05	0.40	0.52	0.44	0.51	0.52	1